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A Proposed Suboptimal Controller for a Bilinear Solar-Assisted Heat Pump System

by

© Hung Quang Le

A THESIS

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Date October 12nd 82

To my mother

and my wife

ABSTRACT

The energy crisis has brought about public concern of the limitation of conventional resources. Suggestions have been made to conserve energy and to seek alternative resources. Solar energy has a promising future to serve as alternative resource especially in the area of space heating.

The controls problem of a solar heating system is shown to have a significant impact on energy use. A new approach to this problem is proposed and studied in this thesis. Thermal as well as economic performance of the system under this new suboptimal controller was also investigated.

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1. INTRODUCTION

1.1 The Importance of Energy.

From the earliest time,Man has tried to meet his increasing demands for goods and services through animal and especially mechanical slaves; tools, machines and engines. Mechanical slaves, in turn, require fuel to operate, and fuel appears in various forms such as coal, oil, uranium, etc.

Fuel or Energy which was abundant and relatively inexpensive is fundamental to present life style; our food supply, our tremendous industry, our conveniences of life, etc., all depend upon a ready supply of energy.

1.2 The Energy crisis.

The heavy dependence of the industrialized countries on oil has made them vulnerable to any happening in the Middle East. The oil embargoes and price increases of the oil cartel OPEC have caused chaos and panic among the developed as well as developing countries. That has ,in turn, triggered public concern about the limitation of the conventional resources.

To solve the problem, it has been suggested not only that more domestic explorations into traditional and alternative sources of energy be promoted but that the public also be taught the importance of energy conservation.

Oilsand projects in Alberta have appeared quite promising in terms of making Canada independent of imported oil for at least into the next century. However, this would be only a short term solution to the "persistent" energy problem, for oil is a limited source of energy.

Is coal able to replace oil in providing us with an undepletable supply of energy? The answer is quite clear. Although coal is considered one of the most abundant resources in North America, it is limited in quantity. For example: The U.S. has an estimated 3.2 trillion tons of coal [1], of which about 217 billion are economically mineable. The U.S. will run out of an economic coal supply in 180 years, if it is mined at the rate of some 1200 million tons per year as proposed by the Federal Economic Agency (FEA).

What about the conservation of energy? It's not a promising solution either, for it's hard to convince people to give up their life styles overnight. Therefore, new sources of energy such as nuclear power and solar energy need to be developed to replace conventional ones.

Nuclear power can certainly satisfy our energy demand but there are still doubts about its safety. The Three-mile Island accident is one of the reasons why people feel concerned. Moreover, it also depends on a finite resource, uranium.

On the contrary, solar energy is a renewable source and poses no potential danger whatsoever to its users. Hence, it seems to be the answer to our persistent problem.

1.3 Solar Energy, a new source of energy.

Looking up at the sky, we see a resource that can supply us with all the energy we need: the inexhaustible, pollution-free energy coming from the sun. How extensive is this resource? Lake Erie receives enough sunlight to provide the total U.S. energy supply.

Before a new source of energy is introduced into our traditional way of life, three conditions need to considered; namely, it should be technically sound and feasible, economically competitive with the conventional sources and socially accepted.

Although solar energy is not yet the answer to our energy problem, it has been proven technically feasible and economically competitive for a few applications. There has been increasing interest in the use of solar energy for heating and cooling, since the amount of energy saved would be considerable. Space heating and water heating account for 22% of U.S. energy consumption. For water heating alone, the amount saved could light every light in the U.S. twice[1].

Solar energy has also been applied, although on a limited basis, to agriculture and industry[1]. It has been projected that about 5% of energy demand for U.S.

agriculture will be supplied by solar energy by 1985. By 2000, the estimated percentage will be 25%, and by 2020 it will become 50%.

In industrial applications such as industrial hot water and drying/dehydration systems, it is estimated that about 0.2% of the energy needed for such applications may be supplied by solar energy by 1985, or about 20% by 2020. Research is also being actively pursued into electrical applications. According to [8], by 1983, Bleyle of America Inc., will receive more than half of its electricity demands from solar energy, and this project is expected to become cost-effective by some time between 1990 and 1995.

Regarding environmental concerns, solar energy has not caused any such serious problems as nuclear power, thus making it socially acceptable; although the occupation of large areas by collectors could be considered detrimental by some owners.

1.4 The heating problem.

Space heating has been recognized as the most favourable area of solar energy applications. In a study reported by Arnold D. Cohen of General Electric[1], 16 million buildings (21% of all U.S.A. buildings) will be equipped with solar heating systems by the year 2000.

It is generally agreed that the way a building is operated has a significant impact on energy use and that a poorly operated building can defeat even the best energy-conserving design. It is also accepted that the use of controls will lead to improved system efficiencies and that the uses of these controls require thorough investigation.

The approach generally used to solve this problem is to model the system and then to find an optimal control strategy for the model.

The active solar heating systems all use the same kinds of components although they might have many different configurations. A solar collector, storage tanks, pumps, a load heat exchanger, and an auxiliary heater are the components that seem indispensable in an active solar heating system.

Each component has a different function[2].

The solar collector is used to collect energy from the sun which is then delivered to the load through the load heat exchanger or stored in the storage tanks. Supplemental energy is provided by the auxiliary heater when solar energy is depleted or insufficient. Pumps operate to carry energy from one part of the system to the other. The successful operation of each component contributes to the successful operation of the whole system. Depending on the desired level of complexity, there are many ways to model those components. According to D.M. Auslander et al[2]:

"...,the load is often modeled as a single 'lump',

> i.e., one temperature characterizes the instantaneous state of the load. The storage tank is also modeled as a single lump. Varying levels of complexity can be used to model the heat exchangers, we have chosen to use single lump models for each of the heat exchangers. All flow components(pumps, piping, valves) have been modeled as static elements. The collector is modeled as a heat exchanger in which the heat input from insolation is a decreasing function of the collector temperature. Any desired function can be used."

The controls problem is considered after every component of the system has been modeled.

The controls problem is concerned with how to control the interaction between these components to lead to a successful design. It is generally agreed that the controls problem consists of a regulation problem and a minimization problem[2].

Auslander has argued that the primary obstacle to developing a method of comparing competing controller designs is due to the lack of general agreement on a uniform standard or specifications.

After rejecting as unreasonable the quadratic performance index which combines temperature deviation from the set point and other factors such as auxiliary energy usage, Auslander went on:

> "Rather than a quadratic performance index, then, it seems reasonable to base controller comparison on some temperature band or minimum temperature... Using the minimum temperature as standard, two competing controllers could be compared by adjusting them so that

> they both have the same minimum temperature over the period of interest. Auxiliary energy usage, or other performance factors, could then be compared directly..."

A good controller is one that not only regulates the system with respect to disturbances, solar heat input and ambient temperature, but also conforms to the minimization standards. According to Winn[3], there are three levels of minimization.

His first level is minimization of the total heat input into the load (not just the auxiliary heat). The second level is minimization of the auxiliary heat used while meeting the regulation specifications. The last level is minimization of parasitic energy.

The second level of minimization is the one that will be used as as minimization standard in this thesis. Reasons for not using the first and third levels have been explained in detail by Auslander et al in [2].

1.5 Summary of past results.

In [3] B. Winn et al. have divided the optimal problem of a solar energy system into three kinds:

> "1. Optimal controllers of the First Kind are represented by those controllers that optimally supply heat to a building in a manner such that a measure of

> the energy supplied and the occupant discomfort is minimized.
> 2. Optimal controllers of the Second Kind represent controllers that maximize the difference between the useful energy collected and the pumping costs associated with collecting the solar energy.
> 3.Optimal controllers of the Third Kind represent controllers that combine the collection and distribution function."

Then, Winn proceeded to find the controllers of the above three kinds. It is the Second Kind controller that has practical significance for the collector-control problem. According to Winn, the solution to the Second Kind problem is a proportional controller. In establishing the objective function of the Second Kind optimal controller, Winn included parasitic losses of the pumps; however, there is some doubt that pumping power should be considered as a loss, since with proper placement of components this power will also be a heat input to the system, thus offsetting a corresponding amount of auxiliary heat input[4].

There have been debates over whether or not the collector pump controller should be on-off or proportional. Before discussing the collector pump controller, it is necessary to describe the equation of the collector. A flat plate solar collector is a device that captures heat from the sun, the useful heat delivered by a solar collector is equal to the energy absorbed by the collector plate less the heat lost to the surroundings. The heat rate input to the system from the collector can be calculated as follows[9]:

$$\dot{Q}_c = A_c(H_T \alpha\tau - U_L(T_p - T_a)) \qquad (1.5\text{-}1)$$

where, $\dot{Q}_c$=useful heat delivered by the collector

A_c= effective collector area (m²)

H_T= incident solar radiation (KJ/hr*m²)

(received on the tilted collector surface)

τ = collector cover transmittance

α = plate absorptance

U_L= collector overall loss coefficient(KJ/hr°Cm²)

T_p= average absorber plate temperature

T_a= ambient temperature

The term $A_cH_T(\alpha\tau)$ is the amount of energy absorbed at the absorber surface. This quantity depends on the effective collector area, the solar radiation, the transmissivity of the glazings, and the absorptivity of the absorber surface. The transmissivity is a function of the glass quality and the angles at which the sunlight hits the glass. Therefore, the glass thickness and the number of the glass sheets are major factors that change the transmissivity coefficient. It is the optical property of the collector surface that determines the absorptivity coefficient. Black surfaces have high absorptivity for the visible range of the solar spectrum. The absorptivity coefficients of carbon black, metal oxides, and black paints often have values above 0.95.

The heat loss from the collector is represented by the term $A_cU_L(T_p - T_a)$.The overall heat loss coefficient, U_L, is in the order of 6 to 11 W/m²°C for one glass glazing and about 4 W/m²°C for two panes.

In equation (1.5-1), T_p represents the average plate temperature, which is difficult to measure. So, instead of using T_p , we use T_i , which is the temperature of the fluid entering the collector, to calculate the heat delivered by the collector.

$$\dot{Q}_c = F\ A_c(H_T\alpha\tau - U_L(T_i - T_a)) \qquad (1.5\text{-}2)$$

where, F is a correction factor, or heat recovery factor. Its value is between 0. and 1. such that $\dot{Q}_c$ calculated by equation (1.5-2) is equal to that evaluated by equation (1.5-1).

If we ignore the heat loss in the pipes between the collector and the storage tank, i.e. the storage tank T_s equals the inlet collector temperature T_i , and if we incorporate the collector pump controller, u_3, into equation (1.5-2), we obtain:

$$\dot{Q}_c = A_C F\ (u_3)(H_T\alpha\tau - U_L(T_s - T_a)) \qquad (1.5\text{-}3)$$

-If on-off controller is used

$F(u_3)=0.$ if $u_3=0.$

$F(u_3)=F$ if $u_3=u_{3max}$

-If proportional controller is used, value of $F(u_3)$ is between 0. and F

To resolve the controversy over on-off and proportional controllers, a comparative study on on-off and proportional controlled system has been reported by Lewis and Carr[5].

The following is results of solar collection in one day of the two controllers.

$\underline{H_T}$	Q_u/Q_T Proportional	Q_u/Q_T On-Off
325 W/m²	0.934	0.955
490.	0.945	0.955
620.	0.947	0.954
701.	0.953	0.954
731.	0.955	0.955

where Q_u= useful energy collected

Q_T= total energy incident on absorber plate.

H_T= solar insolation incident on absorber plate.

Thus, according to Lewis et al.:

> "The slightly lower energy gains made by the proportionally controlled system are due to the low fluid flow rates through the system at low temperature differentials...
>
> While this low flow rate proved to be advantageous at earlier hours by enabling the system to gain energy without excessive cycling, it is now of some disadvantage because the low flow rate holds plate temperature, and, therefore heat losses higher than those encountered in the on/off system."

McDonald et al[6] used an on-off controller on the collector side and used an adaptive optimal algorithm to control other variables.

The approach was basically to identify a linearized model of the system, then to employ optimal control theory to determine gains of the optimal controller which minimize a cost function. This process repeats itself in the next intervals of time. As McDonald put it:

> "The actual building and HVAC system is a non-linear system with operating points which can vary over a wide range. The linearized model is valid only about a region of the operating point; thus, the identification of a linearized model must be an on-going process with optimal controller being modified or adapted for each new linearized model or operating point of the system."

The cost was chosen as the integral quadratic cost functional of plant state variables and control variables. The weighting matrices in the cost functional assign relative importance to state and control variables. Large weights were given to room temperature and auxiliary energy variable in order to maintain comfort conditions and to minimize auxiliary energy variables.

A conventional approach was also presented to compare its result with that of the Adaptive Optimal controller. In Fig.1.1, T_b represents room temperature and T_a is the ambient temperature. If the point representing room and ambient temperature in (T_b, T_a) plane is in the hatched region, only solar energy will be used, if not auxiliary heat will also be used. Simulation results for the heating season are summarized in table 1.1.

Room temperature was shown to maintain slightly more closely to the desired by the conventional controller than the adaptive one. However, the adaptive controller used 28% less auxiliary energy than the conventional due to the fact that storage tank temperatures were kept at lower values , thus making the collector operate more efficiently and collect more solar energy. This approach has once more solidified the role of modern control theory in solar energy heating system ;however, it requires rather sophisticated computations at each update and is inherently limited in its optimization look-ahead time because of the limited duration of accuracy for the linearized model[4]. In this thesis a more direct approach, based on an accurate nonlinear model, is taken.

1.6 Scope of this thesis.

A sub-optimal adaptive controller for a solar-assisted heat-pump system is proposed. Its simulation results are shown to give significant improvement over those of WATSUN program [7](more will be said about the WATSUN approach in chapter three). In view of the results cited above, the collector pump controller is taken to be on-off. Also, following the suggestion of Auslander et al[2], the building loop controller is assumed to be thermostatic. The major control problem remaining is then just the control of the heat pump that is used to upgrade the collected solar energy.

In chapter two, the system models are developed before the optimization problem is addressed. Also presented in this chapter is the derivation of the adaptive preview suboptimal controller. The stability of this suboptimal controller is investigated in chapter three. The simulation results of the suboptimal controller and the WATSUN approach are compared and analysed in later sections of chapter three. In the last chapter, chapter four, the thermal as well as economic performance of the system are examined when major parameters of the system are varied. The viability of the system is also addressed here.

Although the approach can be practically adopted, it is advisable to make a much more thorough investigation before success can be assured. The controller has been designed with the assumption that the weather is a deterministic process, or at least the average ambient temperature of the next day is certainly known. However, the author believes that results will not be very much different even if the next day average ambient temperature cannot be predicted precisely. In other words, it is possible to have similar results even if the process is stochastic in nature.

This controller is believed to be suitable for eventual application even though the work is just exploratory.

TABLE 1.1

	Heating Energy (10^6/BTU)				Aver. room temp.	%saving
	Aux.	solar	Internal	Total		
CONV	3.74	1.05	6.12	10.90	70.01	
ADAP	2.66	2.06	6.12	10.84	69.35	28.8%

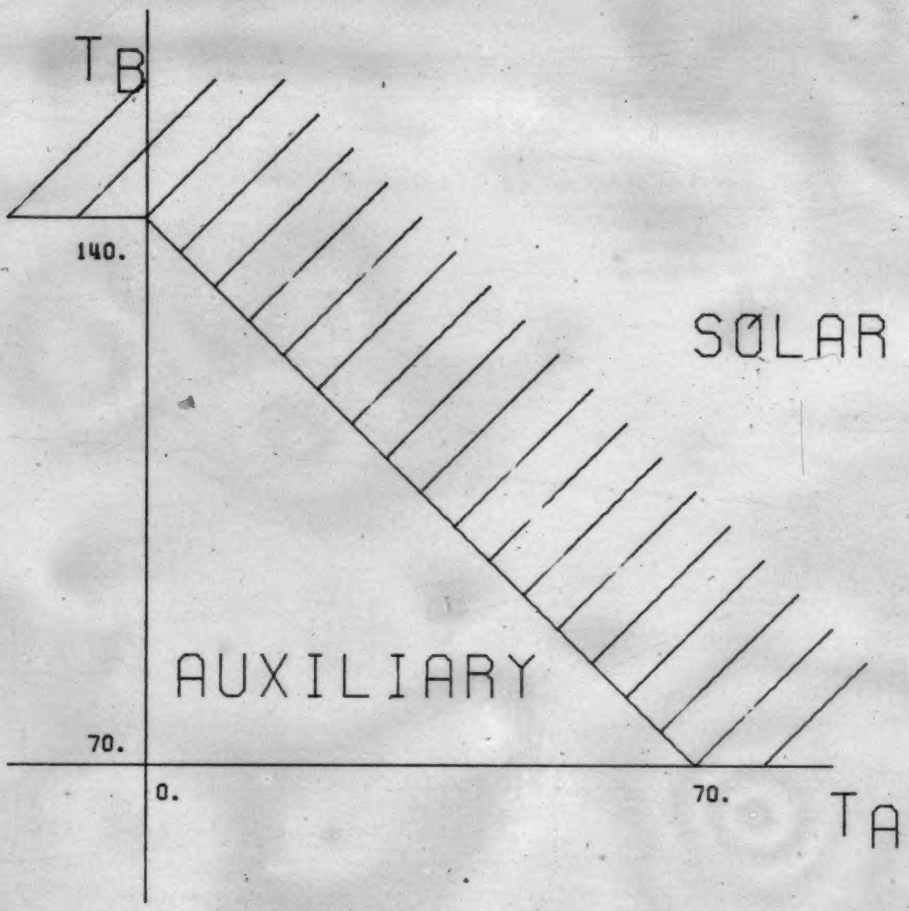
TB
140.
SOLAR
AUXILIARY
70.
0.
70.
TA

Fig.1.1: Conventional Controller schedule for solar energy usage

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2. OPTIMIZATION

2.1 Introduction.

Attention has recently been given to problems of optimal control of heat storage and heat transfer processes in HVAC systems. Control objectives are to minimize the seasonal consumption of purchased energy where solar collection and storage are employed, and /or to minimize the peak-period consumption of electrical energy where off-peak charging are utilized. Among the typical control inputs for, say, a solar assisted heat pump system are mass flow rate through solar collectors, heat pump input power, flow rates to building convectors, and building air-handling variables. State variables are typically the building and storage temperatures.

Potential savings with optimal control seem to be considerable; simulations have shown that improvements of 50% or more are possible under certain conditions with optimal control of solar collector and building variables, as compared with conventional control[1,2]. Better design concepts are needed, however, for the realization of practical, near-optimal feedback controllers for actual HVAC systems. The equations which accurately describe the controlled processes are often nonlinear and one approach has been to linearize them, to periodically update the parameters of the linearized model by relinearizing the nonlinear terms[4] or by using a recursive least squares algorithm to identify parameters from data[2], and then to solve the matrix Riccati equation for near-optimal control in the next time interval. This approach requires rather sophisticated computations at each update and is inherently limited in its optimization look-ahead time because of the limited duration of accuracy for the linearized model.

We propose that a better approach is to explicitly recognize that many of the HVAC processes can be accurately modelled as bilinear processes, where net heat transfer rate is related to the product of pumping rate and temperature difference. This is true for solar collectors, heat pumps, and building heat delivery. Thus, if optimal control can be computed on a basis of a bilinear model that remain globally valid, the need for frequent model updating and reoptimization is eliminated and look-ahead times are not limited by model validity.

The solutions of bilinear control-optimization problems are easily obtained for simple (first order) systems. For a solar heating system with one storage tank, Auslander et al[5] have shown that simple energy considerations dictate that the building-temperature control problem can be decoupled from the collector control problem. Any good regulator that holds building temperature close to its minimum specified level is nearly optimal. Moreover, the optimal collector pump controller is bang/bang (or on/off), when collected solar energy is to be maximized and parasitic (pumping) losses are not subtracted. This follows from the fact that the Hamiltonian is linear in control variable (which is the collector heat removal factor, a monotonic function of pumping rate) and singular arcs can be shown not to exist.

With the same system, but with parasitic losses considered, Winn and Hull [1] have shown that the Hamiltonian becomes convex and pumping rate becomes a continuous function of tank and collector outlet temperatures. There is some doubt that pumping power should be considered as a loss, however, since with proper placement of components this power will also be a heat input to the system, hence will offset a corresponding amount of auxiliary heat input.

A more practical solar heating system for northern lattitudes will probably turn out to be the series solar-assisted heat pump system, however, because of the neccessarily large difference between the building-loop inlet and ambient temperatures. If two storage tanks are included, as shown in Fig. 2.1, the system offers the further attractive option of efficient day-time solar collection with a relatively cool collector feeding a low temperature tank and efficient night-time (offpeak) heat pumping, with a good coefficient of performance, to the high-temperature tank.

The optimal control problem now becomes much more difficult, however, because while the collector and building controls are still decoupled, the optimal heat pump control sequence must depend on both tank temperatures and on the anticipated building heating load and solar intensity patterns.
The formal optimization is difficult, since, while the Hamiltonian is still linear in the controls, it can be shown that singular arcs are possible with this system and, indeed, will be involved in the optimal control sequence. Gunewardana et al [6] have successfully used a numerical search method to compute optimal preview control sequences for a similar nonlinear system.

In the following section we obtain the optimal control sequences by numerical search method. We then postulate that the near-optimal controller could feasibly be implemented with a microprocessor that (1) preselects a set point that depends in a relatively simple way on the weather forecast (2)uses an adaptive controller of which the current value is determined from the previous one (more will be said about this controller in the next chapter)

2.2 The system model.

2.2.1 The heating load.

Using the example 13-2 of [10], the house is assumed to be constructed as follows:

Exterior walls:

- 4" common brick
- 1/2 " plywood
- 2x4 studs
- R-11 insulation
- 1/2 " plasterboard

Floor construction over vented crawlspace

- 25/32 " hardwood finish flooring
- building paper
- 1" plywood sub-floor
- airspace
- R-11 insulation (applied to underside of joists)

Windows: storm windows (4)

Exterior: 1-1/2 " solid core door

Ceiling construction with vented attic space above

- 1/2" plasterboard
- R-19 insulation

The house is 51'x27', and has one wood exterior door 3'x6'-8", and one double glass wood frame sliding patio door with 1/4" air space, 7'x6'-8'. The heat load calculation can be summarized as follows:

	U (Btu/hrft²°F)	A (ft²)	U_b (Btu/hr °F)
Exterior walls	0.07	974	68.18
Windows and sliding patio doors			
double	0.65	47	30.55
single	0.56	207	115.92
Exterior slab doors	0.49	20	9.8
Floors	0.07	1377	96.39
Ceilings	0.05	1377	68.85
Total			389.69

For simplicity, infiltration loss is ignored.

If we designate a_b as the overall heat-loss coefficient of the building, we have:

$$a_b = 389.69 \text{ Btu/hr}^\circ\text{F} = 740.06 \text{ KJ/hr}^\circ\text{C} \qquad (2.2\text{-}1)$$

2.2.2 Collector model.

The collector assumed is a flat plate collector. The rate of solar energy collection $\dot{Q}_C$ is given, as previously, by equation (1.4-3)

$$\dot{Q}_C=A_C F\ (u_3)[H_T\alpha\tau-U_L(T_s-T_a)]$$

Let $S=H_T\alpha\tau$, we have:

$$\dot{Q}_C=A_C F(u_3)[S-U_L(T_s-T_a)] \qquad (2.2\text{-}2)$$

Using the numbers for the example collector given in [1], we have

$A_C=100\ m^2$

$\alpha\tau=0.84$

$F=0.867$

$U_L=12.\ KJ/(hr*m^2*^{\circ}C)$

The collector has to be tilted to optimize the reception of solar radiation, but only radiation data incident on a horizontal surface are available. Hence, the following parameters are needed to process these data in order to obtain values for H_T (see Appendix I).

The collector is placed due south, Hence $\gamma=0^{\circ}$

The location is in Edmonton, of which lattitude is 54°

The tilt slope of the collector is 64°(=lattitude+10°)

2.2.3 The storage tank model.[13].

There are two ways of storing heat, either as sensible heat or as latent heat.

Energy may be stored as sensible heat in liquid (e.g. water) or solid medium (e.g. rock) while selected salts are used to to store latent heat in the form of heat of crystallization. It is sensible heat that has been widely and reliably used, although a lot of research is being done to understand more about the use of latent heat. The amount of energy that can be stored as sensible heat in a storage tank is:

$$\dot{Q}_t = mC_p\Delta T \tag{2.2-3}$$

where, $\dot{Q}_t$=heat capacity of system.

m=mass of storage medium.

C_p=Specific heat of storage medium.

ΔT=Temperature range between which the tank operates.

Losses of sensible heat stored in a tank are due to conduction through the temperature difference between the tank and the surrounding environment.

$$\dot{Q}_l = a_t(T_1 - T_2) \tag{2.2-4}$$

Where, a_t is the heat loss coefficient of the tank

T_1 is the tank temparature

T_2 is the environment temperature

There are two storage tanks in our system, one of which operates at low temperature, and is called the low-temperature tank or low-side tank. The other tank, which works at higher temperature, is called the high-temperature tank or high-side tank.

The low-side tank is used to collect the available solar energy from the collector. It is often operational at a higher temperature than the ambient. By absorbing all available energy of the collector with only small increase in its average temperature, the low-side tank helps keep the average temperature of the collector at lower value than would be the case if the collector were connected directly to high-temperature storage tank, thus increasing the collector efficiency. Moreover, the temperature of the low-side tank, which is higher than the ambient temperature because of the stored solar energy, also helps boost the coefficient of performance of the heat pump.

The energy balance at the low-side tank is :

$$C_c\dot{T}_c = \dot{Q}_c - \dot{Q}_t - \dot{Q}_{hp} \qquad (2.2\text{-}5)$$

where C_c=low-side tank heat capacity,

$\dot{Q}_c$ is the rate at which energy is drawn from the collector. According to equation (2.2-2):

$$\dot{Q}_c = A_c F(u_s)[S - U_L(T_c - T_a)] \qquad (2.2\text{-}6)$$

here T_s is replaced by T_c, temperature of the low-side tank.

$\dot{Q}_t$ is the heat loss of the tank

$$\dot{Q}_t = a_c(T_c - T_a) \qquad (2.2\text{-}7)$$

$\dot{Q}_{hp}$ is the rate at which heat is taken out of the low-temperature tank by the heat pump. An expression for $\dot{Q}_{hp}$ will be found in the next section.

The energy balance at the high-side tank is:

$$C_h\dot{T}_h = \dot{Q}'_{hp} - \dot{Q}'_t - \dot{Q}_b \qquad (2.2\text{-}8)$$

where C_h=high-side tank heat capacity,

$\dot{Q}'_t$ is the heat loss of the tank

$$\dot{Q}'_t = a_h(T_h - T_a) \qquad (2.2\text{-}9)$$

$\dot{Q}_b$ is the heat delivered to the building

$$\dot{Q}_b = \xi u_1 (T_h - T_b) \qquad (2.2\text{-}10)$$

where ξ =heat exchanger coefficient

Q'_{hp} is the heat-pump output (see next section).

According to Lof and Tybout [15], the optimum heat storage is in the range of 50 to 75 Kg of water per one square meter of the collector area. The value of 75 Kg/m² is used here.

2.2.4 Heat pump.

Using a commercially available 3-ton heat pump[14], which is manufactured by the Carrier Corporation, a graph of the COP with respect to the temperature difference between the evaporator and the condenser is plotted in Fig.2.2.

The coefficient of performance can be approximated by a straight line whose equation is:

$$COP(T_h,T_c)=1+(COP_{max}-1)(1-(T_h-T_c)/T_{max}) \qquad (2.2\text{-}11)$$

Where, $COP_{max}=3.5$ and $T_{max}=45.^{o}C$

If we designate u_2 as the heat-pump electrical input, the rate at which heat is taken out of the low-side tank by the heat pump is:

$$\dot{Q}_{hp}=u_2[COP(T_h,T_c)-1.]. \qquad (2.2\text{-}12)$$

The heat pump output is calculated as follows:

$$\dot{Q}'_{hp}=u_2 COP(T_h,T_c). \qquad (2.2\text{-}13)$$

2.2.5 Circulation pumps.

According to experiments, the collector flow rate should be in the neighborhood of 60 liters/hr per one square meter of collector area [12]. Actually, the value of u_s, collector flow rate, is not needed as far as equation (2.2-2) is concerned.

The building loop pump capacity is calculated as follows:

The heat load of the building is:

$$\dot{Q}_l = a_b(T_b - T_a) \text{ KJ/hr} \qquad (2.2\text{-}14)$$

where T_b =building temperature=20 C,

T_a = average ambient temperature.

Take T_a =-12°C [12](average of January)

The maximum heat delivered by the high-side tank is:

$$\dot{Q}_b = \xi u_{,max}(T_h - T_b) \text{ KJ/hr}$$

where ξ=0.8

T_h =average temperature of the high-side tank.

Assume that T_h is approximately equal to 24°C.

In order for the building heat load to be fully met by the heat from the high-temperature tank under these conditions, the building loop pump capacity must be equal to:

$$u_{,max} = a_b(T_b - T_a)/\xi(T_h - T_b)$$

$$= 740.06 \times 32./0.8 \times (24.-20.) = 7400. \text{ Kg/hr}$$

2.3 The Steady-State Optimal Temperatures

We assume that an effective load regulator holds the building temperature constant at a specified T_b using auxiliary heat if necessary. The collector pump rate controller is a bang bang optimal controller:

$$u_3=u_{3max}[1.+\mathrm{sgn}(S-U_L(T_c-T_a))]/2.$$

I.e.,

$$\text{If } \dot{Q}_c=S-U_L(T_c-T_a)>0. \quad u_3=u_{3max}$$

$$\text{If } \dot{Q}_c=S-U_L(T_c-T_a)\leq 0. \quad u_3=0.$$

The dynamic equation for the two tanks are then:

$$C_h\dot{T}_h=-\xi u_1(T_h-T_b)-a_h(T_h-T_a)+u_2 COP(T_h,T_c) \qquad (2.3\text{-}1a)$$

$$C_c\dot{T}_c=-u_2[COP(T_h,T_c)-1]-a_c(T_c-T_a)+A_cF(u_3)[S-U_L(T_c-T_a)] \qquad (2.3\text{-}1b)$$

where the Coefficient of Performance is:

$$COP(T_h,T_c)=1.+(COP_{max}-1.)[1-(T_h-T_c)/T_{max}]$$

and where:

T_h, T_c, T_a, T_b are hot tank, cool tank, ambient, and building temperatures.

C_h, C_c are tank heat capacities

u_1, u_2, u_3 are building loop pump, heat pump, and collector pump rates.

ξ, a_h, a_c are heat transfer coefficients

A_c, F, U_L are collector area, heat removal factor, and loss coefficient.

The required auxiliary heat input rate to the building is

$$\dot{Q}_{aux}=a_b(T_b-T_a)-\xi u_1(T_h-T_b)\geq 0$$

where the equality holds if $u_1 \leq u_{1max}$ can satisfy the equation, i.e. no auxiliary heat is used if the building load can be supplied from storage.

The steady-state optimal problem can be stated as follows:

Minimize $J=a_b(T_b-T_a)-\xi u_1(T_h-T_b)+u_2$

(J represents the total of auxiliary energy and the input power to the heat pump).

subject to:

$$-\xi u_1(T_h-T_b)-a_h(T_h-T_a)+u_2 COP(T_h,T_c)=0 \qquad (2.3\text{-}1)$$

$$-u_2[COP(T_h,T_c)-1]-a_c(T_c-T_a)+A_cF(u_3)[S-U_L(T_c-T_a)]=0 \qquad (2.3\text{-}2)$$

$$-a_b(T_b-T_a)+u_1(T_h-T_b)\leq 0 \qquad (2.3\text{-}3)$$

$$u_1-u_{1max}\leq 0 \qquad (2.3\text{-}4)$$

$$u_2-u_{2max}\leq 0 \qquad (2.3\text{-}5)$$

Establishing Lagrange function and then using the KUHN-TUCKER conditions to solve this problem[9], the steady-state optimal high-side temperature will be proved to be:

$$T_h^o=a_b(T_b-T_a)/(\xi * u_{1max})+T_b$$

T_c^o, u_2^o are found from (2.3-1) and (2.3-2)

Proof:let the Lagrange function be defined as follows:

$L=J+p_1*lhs(2.3\text{-}1)+p_2*lhs(2.3\text{-}2)+p_3*lhs(2.3\text{-}3)$

$+p_4*lhs(2.3\text{-}4)+p_5*lhs(2.3\text{-}5)$

where lhs(2.3-1) is the left hand side of the equation (2.3-1)

lhs(2.3-2) is the left hand side of the equation (2.3-2),etc...

The KUHN-TUCKER conditions are:

$\partial L/\partial T_h=0, \partial L/\partial T_c=0,$

$\partial L/\partial u_1=0, \partial L/\partial u_2=0,$

Constraint equations (2.3-1), (2.3-2), (2.3-3),
(2.3-4), (2.3-5),

p_i *lhs(2.3-i)=0., i=1,2,...,5, and

$p_i \geqslant 0$, i=3,4,5

Applying the KUHN-TUCKER theorem, we obtain the following conditions:

1) $\partial L/\partial T_h=-\xi u_1+p_1[-\xi u_1-a_h-u_2*(COP_{max}-1)/T_{max}]$
$+p_2[u_2*(COP_{max}-1)/T_{max}]+p_3[\xi u_1]=0.$ (2.3-6)

2) $\partial L/\partial T_c=p_1[u_2*(COP_{max}-1)/T_{max}]+p_2*[-u_2*(COP_{max}-1)/T_{max}$
$-a_c-A_cF(u_3)U_L]=0$ (2.3-7)

3) $\partial L/\partial u_2=1+p_1COP(T_h,T_c)-p_2*[COP(T_h,T_c)-1]+p_5=0$ (2.3-8)

4) $\partial L/\partial u_1=-\xi(T_h-T_b)+p_1[-\xi(T_h-T_b)]+p_3[\xi(T_h-T_b)]+p_4$ (2.3-9)
$=0.$ (2.3-9)

5) The constraint equations (2.3-1), (2.3-2), (2.3-3),
(2.3-4), (2.3-5).

6) p_1*lhs(2.3-1)=0 (2.3-10)

7) p_2*lhs(2.3-2)=0 (2.3-11)

8) p_3*lhs(2.3-3)=0 (2.3-12)

9) p_4*lhs(2.3-4)=0 (2.3-13)

10) p_5*lhs(2.3-5)=0 (2.3-14)

11) no restrictions on p_1 (2.3-15)

12) no restrictions on p_2 (2.3-16)

13) $p_3 \geqslant 0$ (2.3-17)

14) $p_4 \geq 0$ (2.3-18)

15) $p_5 \geq 0$ (2.3-19)

Equation (2.3-12) gives:

$p_3 * [-a_b(T_b - T_a) + \xi u_1 (T_h - T_b)] = 0.$ (2.3-12)

Case 1: $p_3 = 0.$ (2.3-20)

Equation (2.3-13) gives: $p_4[u_1 - u_{1max}] = 0.$ (2.3-13)

Case 1.1: $p_4 = 0.$ (2.3-21)

Equation (2.3-14) gives: $p_5[u_2 - u_{2max}] = 0.$ (2.3-14)

Case 1.1.1: $p_5 = 0$ (2.3-22)

From (2.3-20),(2.3-21),(2.3-9),we obtain:

$p_1 = -1.$ (2.3-23)

Substituting (2.3-23) into (2.3-6) and (2.3-7), we find that there are no values of p_2 that can satisfy both (2.3-6) and (2.3-7).

Therefore, this case is excluded.

Case 1.1.2: $u_2 = u_{2max}$ (2.3-25)

This case is excluded for the same reason as the above.

Case 1.2: $u_1 = u_{1max}$ (2.3-27)

Case 1.2.1: $p_5 = 0.$ (2.3-28)

From (2.3-8):

$p_1 = (p_2 * [COP(T_h, T_c) - 1] - 1) / COP(T_h, T_c).$ (2.3-29)

Replacing (2.3-29) into (2.3-7) ,and rearranging,we obtain:

$p_2 = [u_2 * (COP_{max} - 1) / (T_{max} COP(T_h, T_c))] / [-u_2 * (COP_{max} - 1)$
$/ (T_{max} COP(T_h, T_c)) - a_c - A_c F(u_3) U_L].$ (2.3-30)

Equation (2.3-6) is violated when we substitute the values of p_1, p_2, p_3 which are described respectively by equations

(2.3-29), (2.3-30), (2.3-20).

Therefore, this case is also excluded.

Case 1.2.2: $u_2=u_{2max}$. (2.3-31)

Rewriting (2.3-7), we obtain:

$$p_1-p_2=p_2[a_c+A_cF(u_3)U_L]/\ [u_{2max}*(COP_{max}-1)/T_{max}] \qquad (2.3\text{-}32)$$

Because $COP_{max}>0$ and $p_2\geqslant 0$, Hence from (2.3-32)

$$p_1-p_2\geqslant 0. \qquad (2.3\text{-}33)$$

Computing p_5 from (2.3-8),we have

$$p_5=-1-p_1COP(T_h,T_c)+p_2[COP(T_h,T_c)\ -1].$$

Or,

$$p_5=-1-p_2-COP(T_h,T_c)[p_1-p_2] \qquad (2.3\text{-}34)$$

From (2.3-33), (2.3-34) and for $COP(T_h,T_c)>0$, we derive:

$$p_5<0$$

The above condition violates the constraint inequality (2.3-19). Hence, this case is excluded.

Case 2.: $a_b(T_b-T_a)-\xi u_1(T_h-T_b)=0$ (2.3-35)

Case 2.1: $p_4=0$ (2.3-36)

Case 2.1.1: $p_5=0$ (2.3-37)

From equations (2.3-9) and (2.3-36), we obtain:

$$p_3=1+p_1 \qquad (2.3\text{-}38)$$

Replacing (2.3-38) into (2.3-6), we get:

$$p_1[-a_h-u_2*(COP_{max}-1)/T_{max}]+p_2[u_2*(COP_{max}-1)/T_{max}=0 \qquad (2.3\text{-}39)$$

From (2.3-7) and (2.3-39), we obtain:

$$p_1=p_2=0 \qquad (2.3\text{-}40)$$

Replace (2.3-37) and (2.3-40) into (2.3-8)

$$1=0 \text{ (Contradiction)}.$$

This case is excluded.

Case 2.1.2: $u_2=u_{2max}$ (2.3-41)

This case is also excluded, the proof is exactly like that of case 1.2.2.

Case 2.2: $u_1=u_{1max}$ (2.3-42)

Case 2.2.1: $u_2=u_{2max}$ (2.3-43)

This case is excluded for the same reason as that of case 2.1.2 or 1.2.2.

Case 2.2.2: $p_5=0$ (2.3-44)

From (2.3-42) and (2.3-35), as was claimed

$$T_h^0=a_b(T_b-T_a)/(\xi * u_{1max})+T_b. \qquad (2.3\text{-}45)$$

T_c^0, u_2^0 are found from (2.3-1), and (2.3-2) by solving a system of equations. The analysis to find existence conditions for T_h^0, T_c^0 is omitted because the computations are very involved. Moreover, the suboptimal controller is not smart enough to take these conditions into account. This is one area that needs further studies. However, we note that when S equals zero the steady-state optimal temperatures for both tanks do not exist.

The proof is as follows:

since $T_c^0 \geqslant T_a$ (because of stored solar energy),

hence, $S-U_L(T_c^0-T_a) \leqslant 0.$

This implies $F(u_3)=0.$

From (2.3-2), we obtain:

$$u_2^0[COP(T_h^0,T_c^0)-1.]=-a_c(T_c^0-T_a).$$

since, $COP(T_h^0,T_c^0)-1.>0.,$

hence, $u_2^0 \leqslant 0.$

Because u_2 is the heat pump electrical input, it cannot take on negative values.

Hence, $u_2^0=0.$ (2.3-46)

From (2.3-46), (2.3-42), (2.3-45) and (2.3-1), we obtain:

$$T_h^0=(a_h T_a+\xi u_{1max} T_b)/(a_h+\xi u_{1max}). \qquad (2.3\text{-}47)$$

There are no values of T_h^0 that can satisfy both the equations (2.3-45) and (2.3-47).

This completes the proof.

2.3 Optimal control sequence.

Although the steady-state optimal control is of interest in its own right, we are here more interested in the nature of the heat pump control sequences that are optimal under other than steady-state conditions. This is because the solar intensity S and ambient temperature T are never fixed for a long time although they are expected to change gradually. Therefore, the heat pump control will keep changing to move the system operating point toward the new steady-state operating point when S and T change. The optimal values of u_2 calculated as above will not necessarily be optimal under these conditions.

The steepest descent method is used to search for the optimal values of the heat pump control sequence that will minimize a cost index which is a function of auxiliary heat and electrical energy input to the heat pump.

2.3.1 The steepest descent method[8].

The gradient of a function is the vector of partial derivatives of the function, f, with respect to each of its variables. The gradient has a very important property, that is the function value increases at the fastest rate when we move along the the gradient direction. Therefore, the gradient direction is called the steepest ascent direction. Similarly, the negative of the gradient direction indicates the direction of steepest descent.

The steepest descent method is the method for minimizing a cost function that makes use of the direction of the steepest descent. In this method, we start from an initial guess point $\underline{X}_1$ for the argument of the function and iteratively move towards the optimum point according to the formula:

$$\underline{X}_{i+1} = \underline{X}_i + \lambda_i \underline{S}_i$$

where $\underline{S}_i$ is the negative of the gradient of the function f, λ_i is the optimal step size which is obtained by making an one-dimensional search along direction $\underline{S}_i$.

Because the gradient magnitude gets smaller as it is approaching the optimum, the rate of convergence becomes smaller and smaller. In order to improve the convergence characteristics of the steepest descent, the search direction needs to be modified. One of the modified steepest descent methods is the conjugate gradient method which determines the current search direction, based upon the previous direction, the previous gradient, and the current gradient of the function. This method has been shown to converge quadratically, and it will be used to determine the optimal control sequence of the heat pump.

Using the algorithm suggested by Fletcher and Reeves to minimize general functions as follows[8]:

1) Start with an arbitrary point $\underline{X}_1$

2) Set the first search direction

$$\underline{S}_1=-\nabla f(\underline{X}_1)=-\nabla f_1$$

3) Find the point $\underline{X}_2$ according to the relation

$$\underline{X}_2=\underline{X}_1+\lambda_1\underline{S}_1$$

where λ_1 is the optimal step length in the direction $\underline{S}_1$

Set i=2 and go to the next step.

4) Find $\nabla f_i=\nabla f(\underline{X}_i)$ and set

$$\underline{S}_i=-\nabla f_i+(|\nabla f_i|^2/|\nabla f_{i-1}|^2)\underline{S}_{i-1}$$

By trial and error process, the factor multiplying S_{i-1} can be replaced by a constant.

5) Compute the optimum step length λ_i in the direction S_i, and find the new point:

$$\underline{X}_{i+1}=\underline{X}_i+\lambda_i\underline{S}_i$$

6) Test for the optimality of the point $\underline{X}_{i+1}$.

If $\underline{X}_{i+1}$ is optimum, stop the process. Otherwise, set the value of i equal to i+1, and go back to step 4).

The program used to find the optimal path of this system controller is listed in appendix II (the weather conditions used are hypothetical). Specifically, we can divide this program into three parts. The first part of the program is to establish the initial trajectory of the controller. In the second part, the gradient of the objective function is evaluated, and then an one-dimensioned search is performed along the direction determined by the gradient. The search is stopped in part three of the program.

The initial control trajectory is developed as follows: The building loop pump controller u_1 is turned on to such a value that the building heat load is fully met by the heat from the high-side tank. In the case the building load cannot completely be statisfied even with the maximum value of u_1, the rest of the load is supplied from the auxiliary heater. The heat pump controller u_2 is set to such a value that the heat removed from the high-side tank will be replenished by the same amount of heat from the low-side tank. If this cannot be realized, the heat pump is turned on to its maximum value. The collector pump controller is of on-off nature. It is either switched on to the maximum value or switched off completely, depending on whether or not the solar energy collected is more than the heat loss from the collector to the ambient.

Noting that the heat pump controller sequence has 50 sampled values, each of which corresponds to a time interval. In the second part of the program, the gradients are found by imagining that we have a space of 35 dimensions, each represents one out of the first 35 time intervals of the controller time sequence. The gradient vector can be thought of as a 35-dimensioned vector whose each component is calculated by varying the corresponding value of u_2 along that dimension. The last 15 sampled values of u_2 are used to bring the system states back to their initial values, thus preventing the stealth of energy from the final states to save on purchased energy (the objective function).

The new search direction is calculated from the current gradient vector and the old search direction. By experiment, the coefficients of the gradient vector and the old search direction are chosen as -0.5 and 0.5, respectively. Here, the so-called one-dimensioned search actually means searching along one direction, which is determined as above.

The third part establishes the criterion to stop the search.

The optimum control sequence of the heat pump obtained from running the above program is plotted in Fig.2.3[7].

2.3.2 The proposed suboptimal controller.

Using the model, which is developed previously, the numerical optimal path of the heat pump controller sequence was found by the steepest descent method. However, for practical reasons, it is very difficult to implement the optimal controller sequence. Therefore, an implementable suboptimal controller needs to be developed.

Various avenues have been explored to develop a suitable suboptimal controller for this system. One promising approach, which is explained in detail in Appendix V can be be applied to a general bilinear system. However, more work needs to be done before this suboptimal controller can be implemented satisfactorily.

We have also devoted much time to another algorithm. That is, using the constant COP path and the discrimination curve in the state plane of T_h and T_c (Fig. 2.4), the system can be moved from its current operating point to the corresponding steady-state optimum point when the weather changes. For example, the current weather is fairly good and the system operating point is at point A in the state plane (Fig. 2.4). Assuming that the predicted weather is going to be extremely bad, thus, based upon this prediction, a new steady-state optimal point B (Fig. 2.4) is determined. The problem is how and when to get to the new point from the old one. One proposed way is to adjust the heat pump such that its COP (coefficient of performance) is constant, thus moving the system along a constant COP curve in the state plane. The reason for this is to keep the heat pump operating at relatively good values of COP under adverse weather conditions while waiting for the good weather to come. When the discrimination function changes sign, heat pump will be turned to its maximum value to reach the new optimum point (discrimination curve is a curve which passes through the new optimum point, with heat pump set to its maximum). The timing of the heat pump is adjusted such that the new optimum point is reached just when the good weather conditions arrive. But there are doubts that such a trajectory always exists for various weather conditions, let alone being near-optimal. It's also impractical to expect such an abrupt change of the weather. Moreover, the computation will be very much involved, therefore destroying any prospect of its implementation on a microprocessor.

Based upon the results found from running the above program with real conditions of weather, and upon the results of a process of trial and error, we propose a suboptimal preview adaptive controller sequence as follows:

$$u_2(n+1)=0.5*u_2(n)+0.5*K(n)*(1.+T_{hs}-T_h(n)) \quad (2.4.2-1)$$

where,

$u_2(n+1)$=the next-step value of the controller,

$u_2(n)$=the curent value of the controller,

$K(n)$=an adaptive gain changed every step

=ratio of the total high-side's heat loss to the heat pump Coeffient of Performance at step n,

$$=[\xi u_1(n)(T_h(n)-T_b)+a_h(T_h(n)-T_a)]/COP(T_h(n),T_c(n))$$

T_{hs}=The steady-state optimal temperature of the high-side tank (calculated by using equation (2.3-45) with T_a being the predicted average daily ambient temperature).

$T_h(n)$=The current temperature of the high-side tank.

In the following, we will prove that the above suboptimal controller tends to bring the high-side temperatures back to its steady-state optimal values.

Assuming that $$T_h(n)<T_{hs} \qquad (2.4.2\text{-}2)$$

$$u_2(n)>K(n) \qquad (2.4.2\text{-}3)$$

According to definition, K(n) is meant to be the required heat pump electrical input to keep the high-side temperature unchanged (see equation (2.3-1a)).

From (2.4.2-3), we obtain:

$$u_2(n)>K(n)=[\xi u_1(n)(T_h(n)-T_b)+a_h(T_h(n)-T_a)]/COP(T_h(n),T_c(n)).$$

From (2.3-1a) and the above condition, we derive:

$$C_h\dot{T}_h>0.$$

Hence, $$T_h(n+1)>T_h(n)$$

Therefore, the current value of u_2 tends to bring the high-side temperature back to its steady-state optimal value.

On the other hand, if $$T_h(n)<T_{hs} \qquad (2.4.2\text{-}4)$$

and $$u_2(n)<K(n) \qquad (2.4.2\text{-}5)$$

From (2.4.2-1), (2.4.2-4), and (2.4.2-5), we obtain:

$$u_2(n+1)>u_2(n).$$

Hence, as long as (2.4.2-4) holds, values of u_2 will be brought up until u_2 is greater than K (the previous case). If (2.4.2-4) is violated but (2.4.2-5) still holds we go to the next case.

If $$T_h(n)>T_{hs} \qquad (2.4.2\text{-}6)$$

and $$u_2(n)<K(n) \qquad (2.4.2\text{-}7)$$

(2.4.2-7) implies that

$$T_h(n+1)<T_h(n).$$

The current value of u_2 tends to stabilize the system by bringing the high-side temperature back to its steady-state optimal temperature.

For the last case,

if $$T_h(n)>T_{hs} \tag{2.4.2-8}$$

and $$u_2(n)>K(n) \tag{2.4.2-9}$$

From (2.4.2-8), (2.4.2-9), and (2.4.2-1), we obtain:

$$u_2(n+1)<u_2(n).$$

Thus, as long as (2.4.2-8) still holds values of u_2 will be decreased until u_2 is less than K (return to the previous case). However, if (2.4.2-8) is violated while (2.4.2-9) still holds, return to the first case.

In conclusion, regardless of the current values of T and u_2, the suboptimal controller tends to bring the high-side temperature back to its steady-state optimal values. The reason for this is to minimize the auxiliary heat input to the building.

Since, if $$T_h(n)=T_{hs}$$

$$u_1=[a_b(T_b-T_a)]/[\xi(T_{hs}-T_b)]=u_{1\max}$$

So, $$\dot{Q}_{aux}=a_b(T_b-T_a)-\xi u_{1\max}(T_{hs}-T_b)=0.$$

However, if more information on T_c is taken into account so that u_2 is minimized at the same time with $\dot{Q}_{aux}$, there can be further improvement on the system performance. The approach described by Appendix V may be useful in doing so. The stability and the near-optimality of this controller are fully explored in the next chapter.

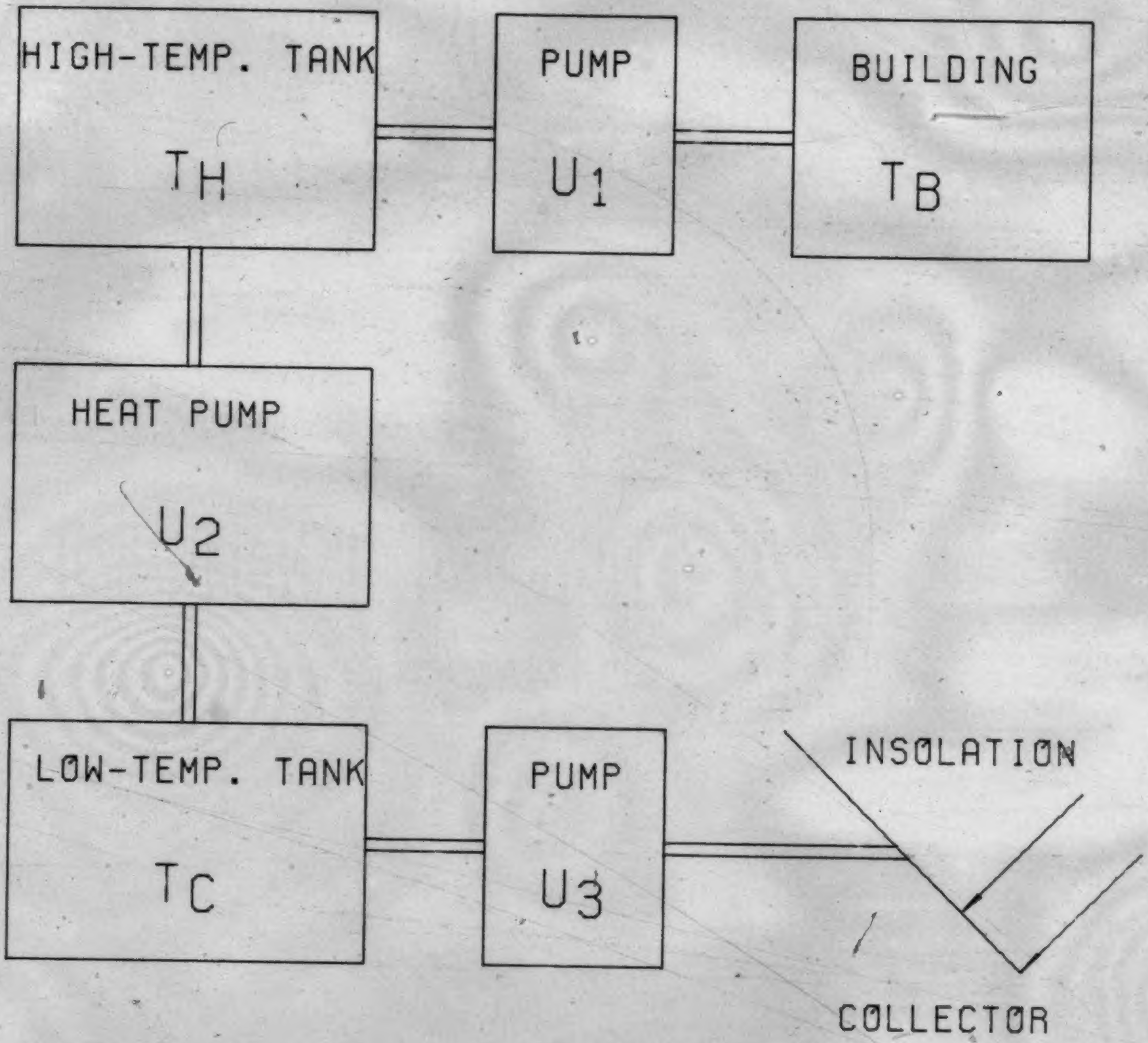
HIGH-TEMP. TANK
T_H
PUMP
U_1
BUILDING
T_B
HEAT PUMP
U_2
LOW-TEMP. TANK
T_C
PUMP
U_3
INSOLATION
COLLECTOR

Fig.2.1: The System Configuration

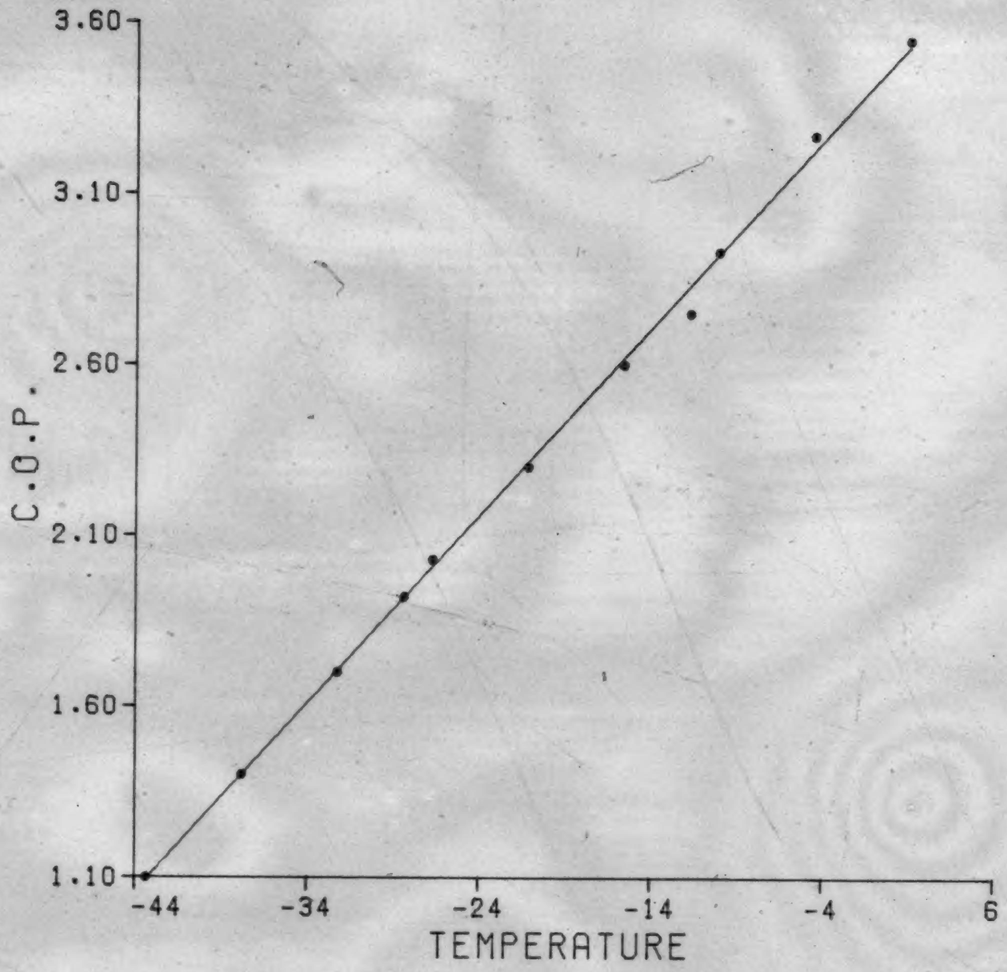

Fig.2.2: Heat Pump Coefficient

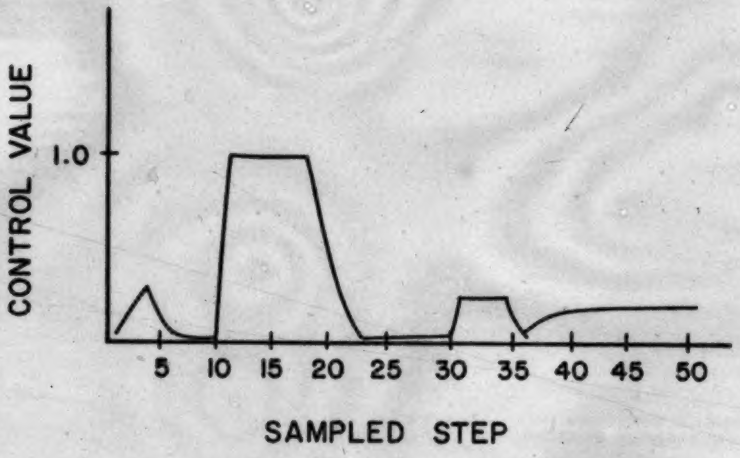

Fig.2.3: Optimum Control Sequence

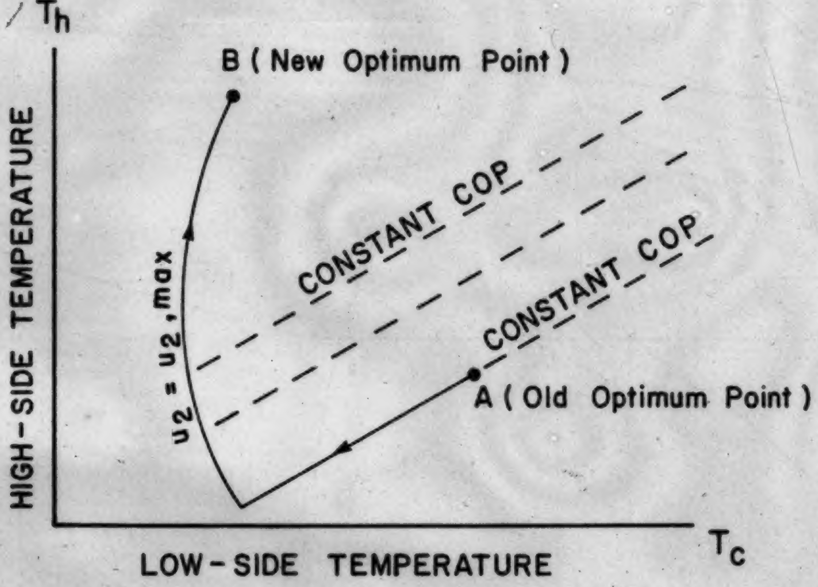
T_h
B (New Optimum Point)
CONSTANT COP
CONSTANT COP
$u_2 = u_{2,max}$
A (Old Optimum Point)
HIGH-SIDE TEMPERATURE
LOW-SIDE TEMPERATURE
T_c

Fig.2.4: State Plane

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3. SIMULATION and THE SUB-OPTIMAL CONTROLLER

3.1 The stability of the suboptimal controller.

For a control system, stability is often considered as one of the most important criterion to be investigated. There are many methods such as the Nyquist, the Routh-Hurwitz, the Lyapunov method,etc., that can be employed to analyse the stability performance of a system. In the following, the Lyapunov method will be used to examine the stability of the solar-assisted heat pump system.

The dynamic equations of the system are as follows:

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + (\underline{B}\,\underline{x} + \underline{C})u_2 + (\underline{D}_1\,\underline{x} + \underline{D}_2)u_1 + \underline{E}\,F(u_3) + \underline{H} \qquad (3.1\text{-}1)$$

where

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} T_h \\ T_c \end{bmatrix}$$

$$\underline{A} = \begin{bmatrix} \frac{-a_h}{c_h} & 0 \\ 0 & \frac{-a_c}{c_c} \end{bmatrix} = \underline{A}^T$$

$$\underline{B} = \frac{-(COP_{max}-1)}{T_{max}} \begin{bmatrix} \frac{1}{C_h} & \frac{-1}{C_h} \\ \frac{-1}{C_c} & \frac{1}{C_c} \end{bmatrix}$$

$$\underline{C} = \begin{bmatrix} \dfrac{COP_{max}}{C_h} \\ \\ \dfrac{1-COP_{max}}{C_c} \end{bmatrix}$$

$$\underline{D}_1 = \begin{bmatrix} \dfrac{\xi}{C_h} & 0 \\ \\ 0 & 0 \end{bmatrix} \qquad \underline{D}_2 = \begin{bmatrix} \dfrac{\xi T_b}{C_h} \\ \\ 0 \end{bmatrix}$$

$$\underline{E} = \begin{bmatrix} 0 \\ \dfrac{A_c[S-U_L(T_c-T_a)]}{C_c} \end{bmatrix}$$

$$\underline{F} = \begin{bmatrix} \dfrac{a_h\ T_a}{C_h} \\ \\ \dfrac{a_c\ T_a}{C_c} \end{bmatrix}$$

Equation (3.1-1) has the form:

$$\dot{\underline{x}} = \underline{f}(\underline{x},u_1,u_2,F(u_3)) \qquad (3.1\text{-}2)$$

In the neighborhood of the steady-state optimal point, define:

$$\delta\underline{x} = \underline{x} - \underline{x}_{opt} \qquad (3.1\text{-}3)$$

$$\delta u_1 = u_1 - u_{1,opt} \tag{3.1-4}$$

$$\delta u_2 = u_2 - u_{2,opt} \tag{3.1-5}$$

$$\delta F(u_3) = F(u_3) - F(u_{3,opt}) \tag{3.1-6}$$

where $\underline{x}_{opt}$, $u_{1,opt}$, $u_{2,opt}$, $u_{3,opt}$ are the steady-state optimal values of $\underline{x}$, u_1, u_2, u_3, respectively.

Substituting (3.1-3), (3.1-4), (3.1-5), (3.1-6) into (3.1-2), we obtain:

$$\delta\dot{\underline{x}} = \underline{f}(\underline{x}_{opt} + \delta\underline{x},\ u_{1,opt} + \delta u_1\ ,\ u_{2,opt} + \delta u_2,\ F(u_{3,opt}) + \delta F(u_3))$$

Linearizing the above equation around the optimum steady state point, and retaining only the first-order terms, we get:

$$\delta\dot{\underline{x}} = \underline{f}(\underline{x}_{opt},\ u_{1,opt},\ u_{2,opt},\ F(u_{3,opt})) + \left.\frac{\partial\underline{f}}{\partial\underline{x}}\right|_{opt} \delta\underline{x} \ +$$

$$\left.\frac{\partial\underline{f}}{\partial u_1}\right|_{opt} \delta u_1 \ + \left.\frac{\partial\underline{f}}{\partial\ _2}\right|_{opt} \delta u_2 \ + \left.\frac{\partial\underline{f}}{\partial F(u_3)}\right|_{opt} \delta F(u_3) \tag{3.1-7}$$

where, $(\ldots)\Big|_{opt}$ indicates the expression (...) is evaluated at the optimum point.

From (3.1-1), we have:

$$\left.\frac{\partial\underline{f}}{\partial\underline{x}}\right|_{opt} = \underline{A} + \underline{B}\ u_{2,opt}$$

$$\left.\frac{\partial \underline{f}}{\partial u_1}\right|_{opt} = \underline{D}_1 \, \underline{x}_{opt} + \underline{D}_2.$$

$$\left.\frac{\partial \underline{f}}{\partial F(u_3)}\right|_{opt} = \underline{B} \, \underline{x}_{opt} + \underline{C}$$

$$\left.\frac{\partial \underline{f}}{\partial F(u_3)}\right|_{opt} = \left.\underline{E}\right|_{\underline{x}_{opt}}$$

Since, $\underline{f}\,(\underline{x}_{opt}, u_{1,opt}, u_{2,opt}, F(u_{3,opt})) = \dot{\underline{x}}_{opt} = 0.$, (3.1-7) can be rewritten as:

$$\delta\dot{\underline{x}} = (\underline{A} + \underline{B}\, u_{2,opt})\, \delta\underline{x} + (\underline{D}_1 \, \underline{x}_{opt} + \underline{D}_2)\, \delta u_1 + (\underline{B}\, \underline{x}_{opt} + \underline{C})\, \delta u_2 + \left.\underline{E}\right|_{\underline{x}_{opt}} \delta F(u_3) \qquad (3.1\text{-}8)$$

Here, $\delta F(u_3)$ can in principle depend on $x_{2,opt} = T_c^{\circ}$, since $F(u_3)$ is a bang-bang controller which changes its value when $S - U_L(T_c - T_a) = 0$.

If, for given S and T_a, T_c° does not satisfy the above switching condition, i.e. $S - U_L(T_c^{\circ} - T_a) \neq 0$, there will be a neighborhood of T_c values which do not satisfy it either. Hence, $\delta F(u_3) = 0$ in the neighborhood of $\underline{x}_{opt}$.

However, if, for given S and T_a, T_c° satisfies the switching condition, i.e. $S-U_L(T_c^\circ-T_a) = 0$, we have:

$$\underline{E}\Big|_{\underline{x}_{opt}} = \begin{bmatrix} 0 \\ \dfrac{A_c[S-U_L(T_c^\circ-T_a)]}{C_c} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \underline{0}$$

In short, $E\Big|_{\underline{x}_{opt}} \delta F(u_3)$ is always zero.

So, (3.1-8) becomes:

$$\delta\underline{\dot{x}} = (\underline{A} + \underline{B}\, u_{2,opt})\delta\underline{x} + (\underline{D}_1\, \underline{x}_{opt} + \underline{D}_2)\, \delta u_1 + (\underline{B}\, \underline{x}_{opt} + \underline{C})\delta u_2. \qquad (3.1\text{-}9)$$

According to Appendix IV, the homogeneous system

$$\delta\underline{\dot{x}} = (\underline{A} + \underline{B}\, u_{2,opt})\delta\underline{x} \qquad (3.1\text{-}10)$$

has negative eigenvalues for any given values of $u_{2,opt}$. Hence, the system described by equation (3.1-10) is asymptotically stable [3]. System (3.1-9) reduces to system (3.1-10) when $\delta u_{\substack{1\\2}} \rightarrow 0$. Hence, according to [4], the system described by (3.1-9) is also asymptotically stable for small values of $\delta\underline{x}$ and δu_1, δu_2.

3.2 The near-optimality characteristics.

In order to prove that the suboptimal controller is really suboptimal, we used the steepest descent method to find the optimal control sequence of the heat pump with an initial trial sequence which is the same as that of the suboptimal controller. Results of the search (in Fig.3.1) show that the suboptimal controller sequence is very close to that of the optimal controller, and the costs associated respectively with these two controllers are also close (the period involved is one day).

3.3 Simulation and design methods.

Simulation has become a powerful tool in developing as well as designing solar processes. TRNSYS program has been widely used in obtaining information for various solar systems. Programs like f-chart have been used by manufacturers, and others to design solar heating and cooling systems. The reason why simulation is so popular in exploring solar processes is that physical experiments are expensive, time-consuming, and generally not repeatable. However, simulation can provide us with much information regarding response and design aspects of the system. As John Duffie [1] put it:

> "Properly formulated, simulations can provide much of the same thermal performance information as physical experiments and require orders of magnitude less time

> and expense. Effects of process design variables can be studied systemmatically. Thermal performance, with cost information, allows determination of least cost systems when process design parameters are varied."

However, there are some problems confronting the simulator of a solar process. For instance, one has to ask what level of detail is appropriate for a simulation? What kind of collector model should be used? If more than necessary details are included, computer time is wasted. On the other hand, information obtained could be misleading for lack of accuracy in modelling the system components. Meteorological data is also a problem, as Duffie put it:

> "...The limited number of locations also leaves many simulators without data for locations of interest to them, and there are still uncertainties in the calculation of radiation on tilted surfaces from data on horizontal surfaces."

Duffie summarized his remarks concerning simulations as follows:

> "We have at our deposal increasingly powerful and useful tools for understanding and designing solar process systems. Simulations can produce for us information which can not be obtained in any other pratical way.
>
> The users of simulation programs, however, must be aware of both the capabilities and the limitations of the programs they are using. Misuse of simulation can produce more information on system in less time than any other method! Users should excercise their engineering judgement in the use of simulation based on the best possible knowledge of the science and engineering of the solar processes they are

simulating."

The simulation is done with the set of weather data for Edmonton from the first of October, 1967 to the first of April, 1968. These data which are collected by Environment Canada, are available only in hourly measurements; therefore, in our simulation, we assume the weather data are unchanged during each hour. We also assume that the average ambient temperature of the day is known one day in advance. A listing of the simulation program is given in Appendix III. There are six subroutines in this program. To simulate the suboptimal approach, we need five subroutines, which are READIN, WEATHR, AVERA, OPSET, and SYSANA. The WATSUN approach is simulated through the use of subroutines READIN, WEATHR, and SYSWAT. Subroutines SYSWAT and SYSANA have their own subroutines TEMP1 and TEMP, respectively.

Subroutine READIN is used to read in the parameters of the system. Subroutine WEATHR, which is taken from the WATSUN 2 program, is used to process meteorological data, as explained in the Appendix I. AVERA is used to calculate the daily average temperatures that are to be used in evaluating the adaptive preview suboptimal controller (see section 2.3.2). Subroutine OPTSET computes the optimal steady-state temperatures of the high-temperature tank, these values will be used as the set points in the suboptimal controller. SYSANA, with the help of TEMP, carries out the simulation with information from subroutines WEATHR, AVERA, and OPTSET. The simulation of the WATSUN approach is done by subroutine SYSWAT.

The Euler integration method is used with the sampling time of 15 minutes, or 4 samples per hour. Comparing the bulding heat loads calculated respectively for the case of 4, 5, 6 samples per hour (table 3.1), a small error of 0.7% is reported. This justifies the use of 4 samples per hour.

3.4 Results of the WATSUN approach.

The strategy of the WATSUN approach is as follows[2]:

1)Check whether or not collection is possible. If no, go to the next step. If yes, is the storage tank less than a preset maximum temperature? If yes, tank S1 is provided with solar collected energy, if no, the collected energy is dumped.

2)Check whether or not heating demand exists? If no, go to the next step; If yes,can tank S1 meet all the demand?

-If yes, S1 supplies heat to the load and go to the next step.

-If no, can tanks S1 and S2 meet all the demand? If no, auxiliary heat is used to supplement S1 and S2.

3)Can the heat pump work with the existing temperatures of tanks S1 and S2. If no, return to step 1; If yes, heat pump takes heat from tank S1 and supplies it to tank S2.

The heat pump used here is a 3-ton Carrier heat pump, as mentioned earlier (chapter 2). For lack of specific data, we assume that there is no temperature limit on the heat pump's range of operation except with T_{max} as an upper limt. The heat taken from tank S1 to supply to S2 (step 3) is approximated by equation (2.2-12) with u_2, the heat-pump electrical input, approximated by the following equation (see Fig.3.2):

$$u_2=2.+(T_c-T_h+44.)/25.\ \text{kW} \qquad (3.4-1)$$

and

$$u_2<W_{max}=3.8\ \text{kW}.$$

3.5 Results of the suboptimal controller.

To use the suboptimal controller approach, we assume that the heat pump electrical input, u_2, can be changed at will.

The strategy for the control of the collector pump, heat pump and building circulation pump is as follows:

1)The collector control is bang-bang, i.e., whenever there is more energy to be collected than to be lost to the surroundings, the collector pump is turned on to the maximum, otherwise, it is switched off.

2)The heat pump controller is based upon the suboptimal controller sequence which was mentioned earlier.

3)The building circulation pump is used whenever necessary to keep the building temperature at a pre-determined value. When the heating load is not fully met by the storage tank, auxiliary heat will be used.

4)If the low-side tank temperature is above a preset upper limit, it indicates a mild weather condition. Under this special condition, the heat pump will be turned off and a mass exchange between the two tanks instituted, thus effectively making the system a one-tank pure solar system.

The simulation period is 150 days long. It is from October 1, 1967 to April 1, 1968. This period can be roughly divided into three smaller periods. The first period is from the 1st day to the 80th day. The second period is from the 81th day to the 130th day, and the third period includes days of the rest of the simulation period (see Fig.3.3 and Fig.3.4). The simulation results of the WATSUN and the suboptimal approaches are plotted in Fig.3.5 and 3.6 which show that the energy consumed by the WATSUN approach is more than that of the suboptimal controller for the first period, and for the third period. In the first period, the weather is considered good by the suboptimal controller, hence, strategy 4 is taken. Therefore, in this period, the saving of purchased energy is mainly due to the doubling of the mass storage of the suboptimal controller. In the following, we will look at the performances of the two approaches only in the second and the third period (Fig.3.7 and Fig.3.8) to see whether the suboptimal controller consumes more or less energy in less favourable conditions of weather. In the third period of interest, 130th day to 150th day, the weather is relatively better than the second period (but worse than the first), and the suboptimal controller (using strategy 2) makes a saving of 10% in purchased energy ,as seen in Fig.3.7(the total saving for the entire time, 150 days, is 31%). Fig.3.7 also shows that in the cold period, 80th day to 130th day, the suboptimal controller and the WATSUN approach consume almost the same amount of purchased energy. In other words, there is no benefit using the suboptimal controller when the weather is so bad. If we increase the collector area from 100 to 300 m², using strategy 2, 37% saving of purchased energy is obtained (Fig.3.8). But most of these savings come from the third period. This ,once more, confirms the above conclusion that unless the weather is very bad the suboptimal controller always outperforms the WATSUN algorithm. It should also be noted that the second period is for the months of December and January which, for Edmonton, are not likely to be suitable for solar heating in any case.

Table 3.1

	samples per hour		
	4	5	6
Building load(GJ)	69.39	69.34	69.33

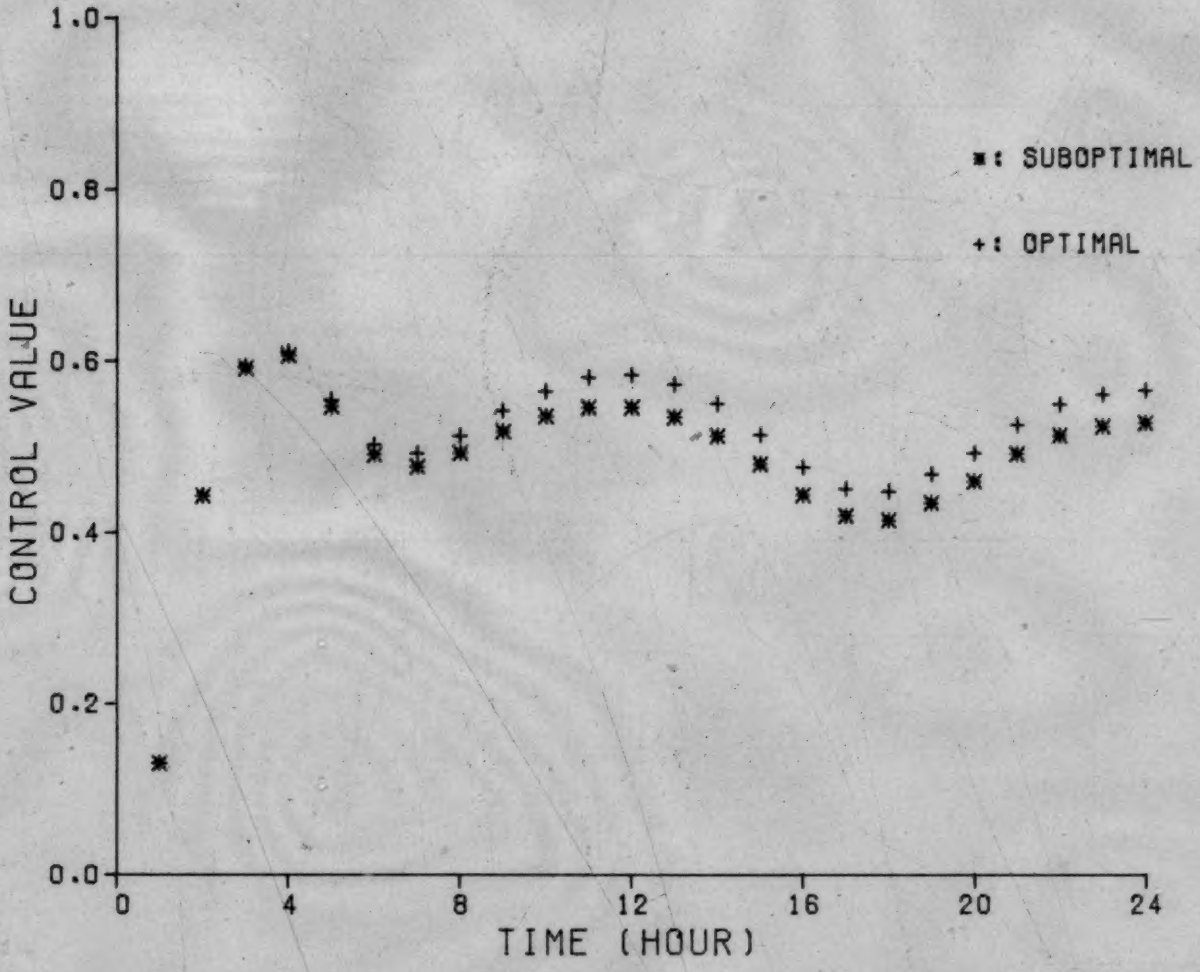

Fig.3.1: Suboptimal and Optimal Controller Values

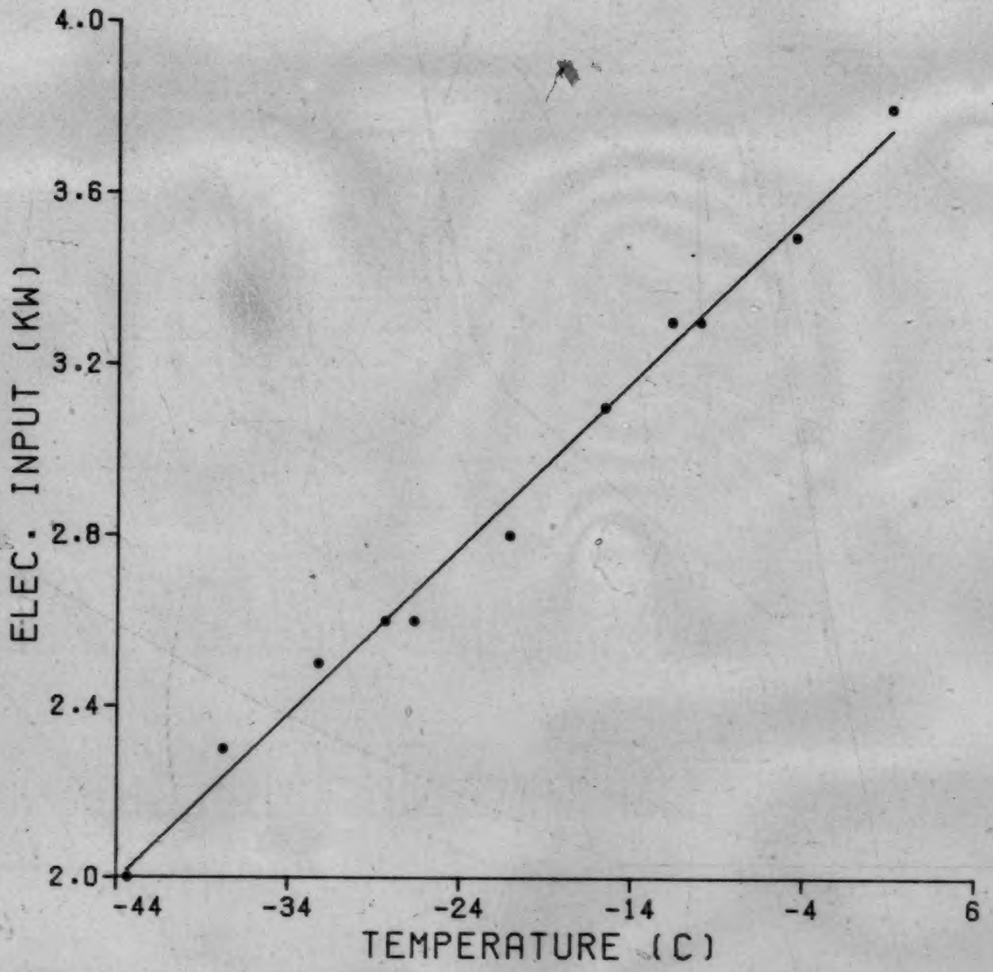

Fig.3.2: Heat Pump Electrical Input

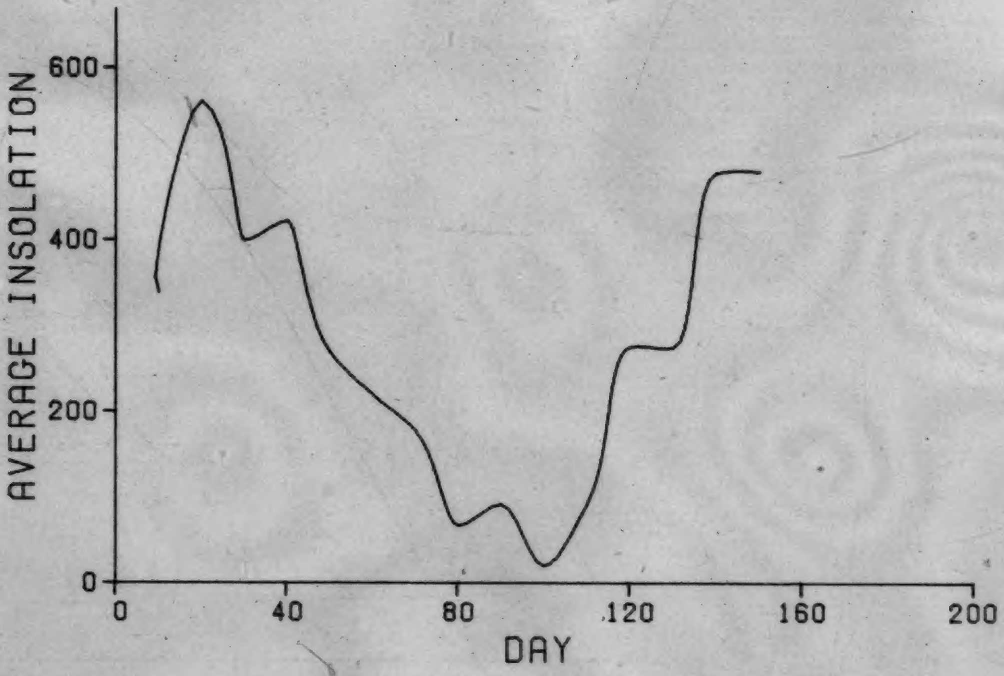

Fig.3.3: Average Insolation

(per ten days)

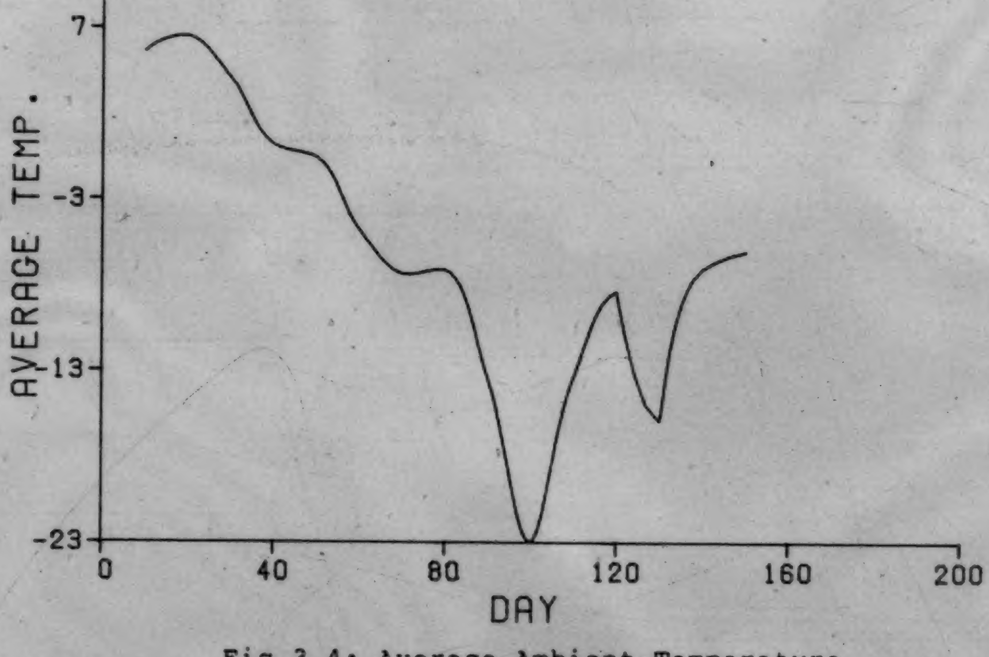

Fig.3.4: Average Ambient Temperature

(per ten days)

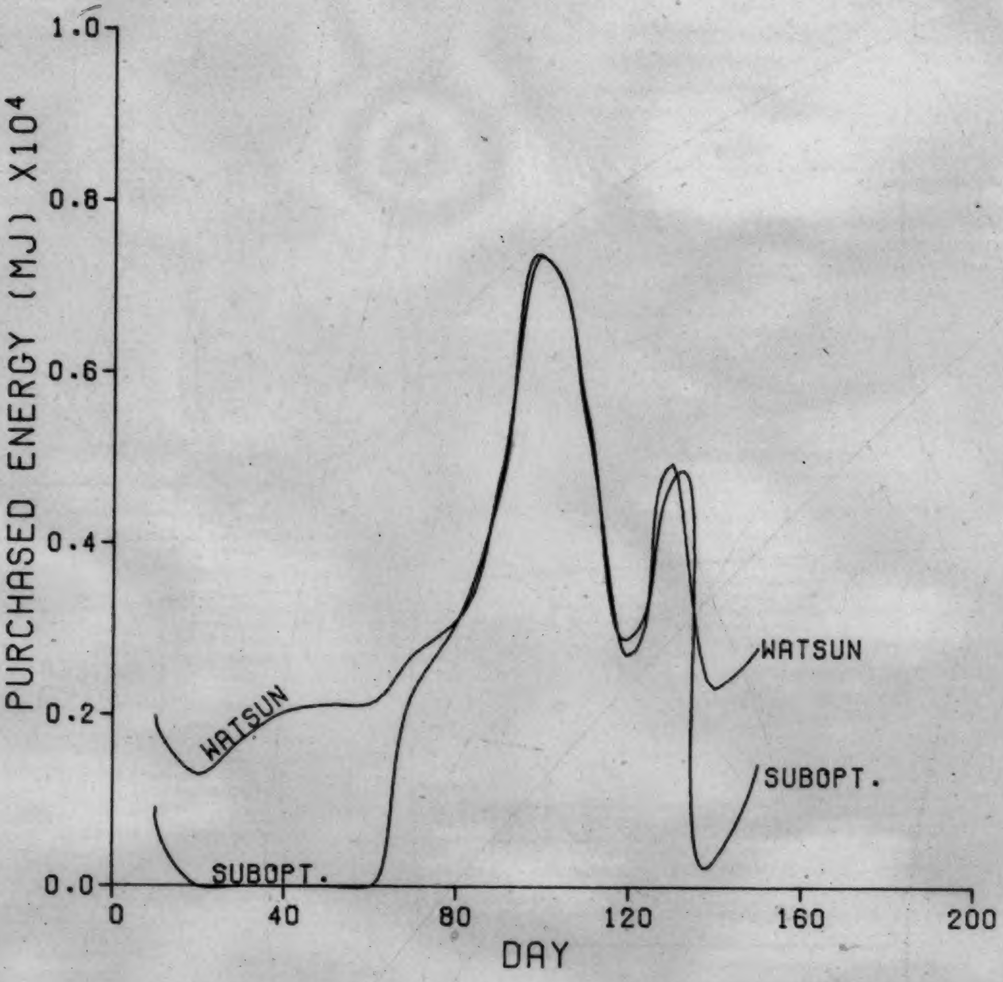

Fig.3.5: Simulation results

(collector area=100 m²)

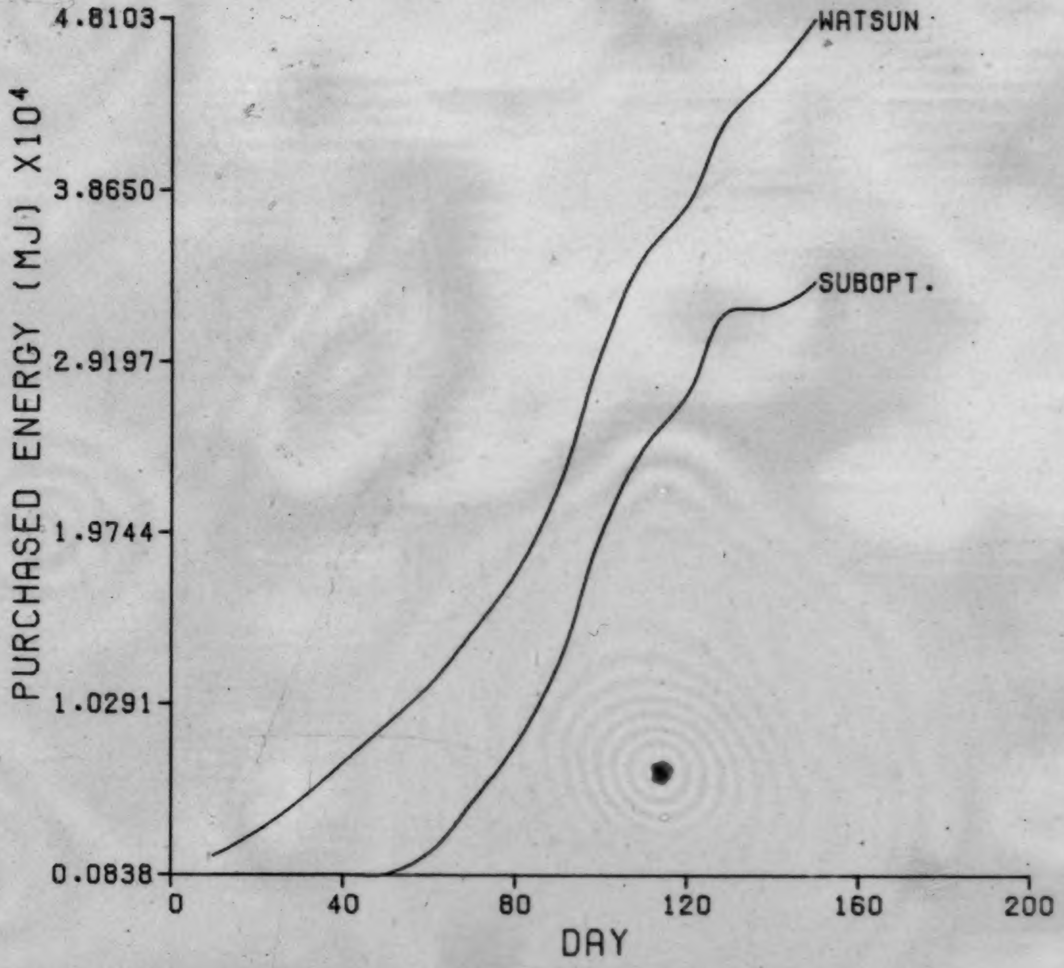

Fig.3.6: Simulation results

(cummulative energy cost, collector area=100 m²)

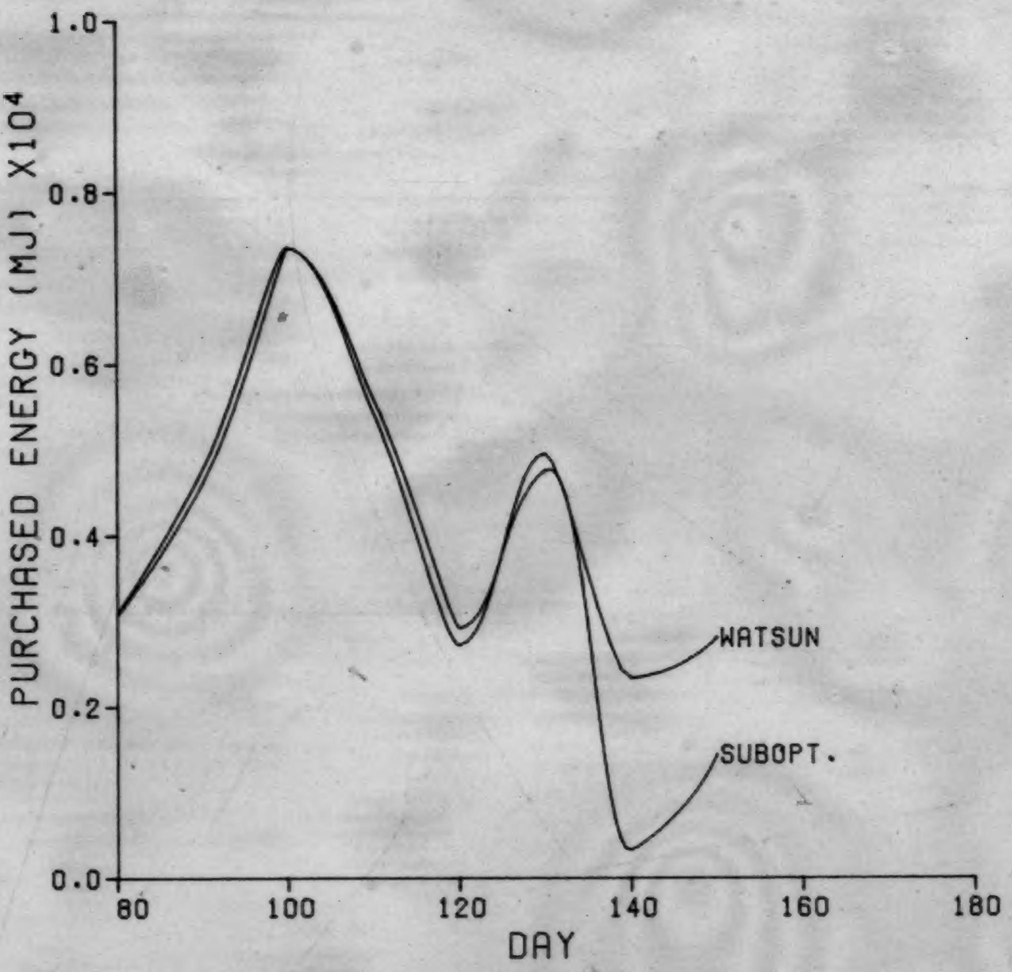

Fig.3.7: Simulation results

(2nd and 3rd periods, collector area=100 m²)

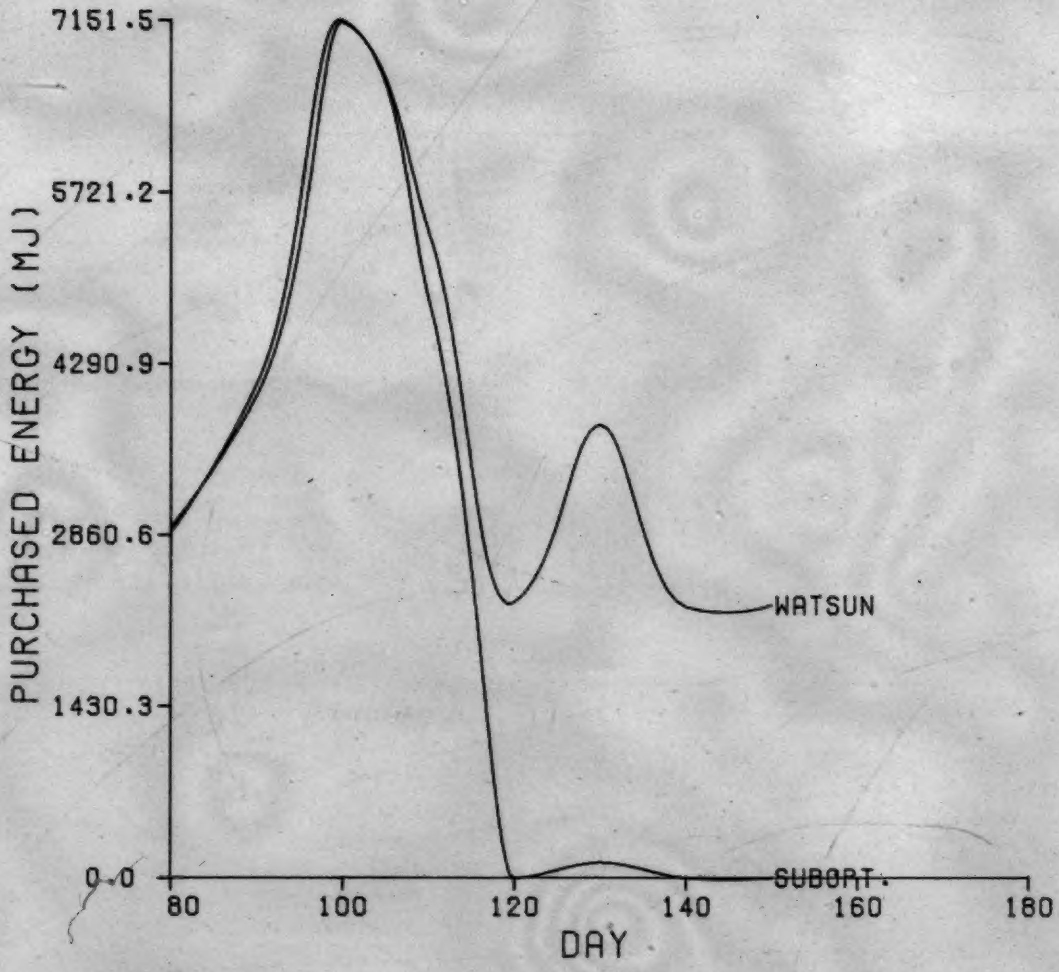

Fig.3.8: Simulation Results

(collector area=300 m²)

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4. THERMAL and ECONOMIC ANALYSIS

4.1 Thermal sensitivity analysis.

The performance of a solar heating system depends on many factors, some of which are beyond our control, e.g. the ambient temperature or the insolation. But there are possibilities to improve the performance of a system. For example, it has been shown that the system efficiency can be enhanced with the proper sizing of system components. The purpose of this study is to identify areas for possible performance improvement and (or) cost reduction through utilization of thermal and economic models.
The saving percentage is defined as the percentage of the system heat load which is provided by the solar energy; or specifically, it is equal to ,in percent, the difference between the total building heat load and the purchased energy, divided by the total building heat load.

4.1.1 Mass storage.

For a given collector area, various values of the storage mass per unit collector area were used to calculate the savings percentage (it is assumed that the tanks are always of equal mass). Using simulations as described in the previous chapter, with the same particular set of weather data, each value of the storage mass was used to find the corresponding savings percentage (Fig. 4.1). In Fig.4.1, we note that when the collector area is small, the system performance becomes worse as the storage mass per unit collector area increases from 40. to 100 Kg per one square meter of collector area. The phenomenon is reversed when the collector area is large. This is due to the fact that if the tank storage mass is larger, the average tank temperature becomes lower, and this leads to a worse heat-pump COP, but to a better collector efficiency. Hence, it follows that better performance of the collector does not compensate for a worse COP in the case of a small collector. The reverse is true for a large collector, because if a collector is large enough, solar collection would out-perform the heat pump[1].

4.1.2 Collector area.

For a given storage capacity per unit collector area, the savings percentage is directly proportional to the collector area (Fig4.2). A larger collector helps capture more solar energy, and therefore more energy can be stored to be sent to the load later.

4.1.3 Heat exchanger coefficient.

The heat exchanger coefficient, which is positive and always less than or equal to unity, can cause a difference of 25% in the saving percentage if the coefficient is increased to 1. from its lower value of 0.1 (Fig.4.3). The heat exchanger is used to transfer heat from the liquid inside the duct to the room airspace. A higher heat exchanger coefficient means a larger amount of heat to be transferrable and a less amount of auxiliary heat to be needed.

4.1.4 Collector covers.

Using the numbers in [5], the savings results of the system with zero-, single-, and doubled-glazed are tabulated in table 4.1. The zero-glazed is always out-performed by the single-, and the doubled-glazed. For large collector, the double- and the single-glazed save almost the same amount of energy, but for smaller collector, the single-glazed can cause a difference of 0.16% in saving of purchased energy, therefore, the single-glazed colector is the most appropriate. This agrees with the conclusion of [6].

4.1.5 Building loop pump capacity.

Supposing that we encounter a scenario as follows: there is a large instantaneous heating demand, which the high-side tank is capable of meeting, but the building loop pump capacity is not sufficient to transfer fully to the room the amount of energy needed. In that case, auxiliary heat must be used to keep the room at desired temperature, thus lowering the saving percentage, as can be seen in Fig.4.4. However, the building loop pump only aids in transferring heat to the load on the condition that the tank stores enough energy and a heating demand exists; therefore, there is a limit on the increase of the saving percentage when the loop pump capacity is increased. This explains the saturation branch of the curve in Fig.4.4. Approximately 16% more of saving percentage is recorded here. An effective collector area of 20 m², a mass storage of 75 Kg per unit collector area (m²), and a heat exchange coefficient of 1. were used to plot fig.4.4.

4.1.6 Tanks heat loss coefficient.

The change of the tanks heat loss coefficient(a_c and a_h) does not cause a significant change in the performance of the system. Less than 2% difference of the saving energy is found when the heat loss coefficients are changed from 5 to 15KJ/hr^{o}C (Fig.4.5).

4.1.7 Tilt slope.

A variation of the tilt slope could result in a worse or a better saving percentage. In Fig.4.6, the optimum slope is found to be in the neighborhood of the region lattitude. This agrees with the rule of thumb that the optimum tilt slope is in the range from (latitude-10^{0}) to (latitude+10^{0}).

4.1.8 Heat pump capacity.

The optimum value of the heat pump capacity is around 5kW (Fig.4.7). The savings percentage is slightly lower if the heat pump capacity is increased beyond 5 KW. This is understandable if we we note that in our simulation time, there are fourteen days, the 93rd day to the 106th day, without sunlight. Hence, in this period, the steady-state optimal temperatures of the two tanks do not exist (Chapter II), though we can still calculate the pseudo-optimal values of the high-temperature tank by equation (2.3-45). Therefore, any successful efforts by the heat pump to reach these pseudo-optimal values will result in more consumption of purchased energy. When the heat pump capacity is less than 5 KW, the system will not be able to reach the pseudo-optimal points. The reverse is true for systems with higher heat pump capacity. However, if we shut off the heat pump completely during the cold period this will degrade the system performance. The reason is that the heat pump will work with bad values of the COP (because of low values of the low-side temperature) when the weather becomes milder, thus decreasing the saving percentage. This is another area where further work is needed to improve the performance of the suboptimal controller.

4.2 Economic analysis.

The thermal performance of the system was dealt with in the preceding discussion, but the real impact on the part of the designer lies on the question of cost. No matter how good the system performance is, it will not be realizable if the cost is unpractically high. According to J.Duffie and W.A.Beckman[2], the annual cost of a solar heating system consists of many elements such as the annual cost of ownership, the annual cost of operation, and the yearly cost of maintenance. As Duffie and Beckman put it:

> "The major annual cost of a solar heating system, without auxiliary energy, includes: the annual cost of operating the system; the cost of power for the pumps, blowers, and so on; and the yearly cost of maintenance. The annual cost of ownership includes cost associated with the initial investment, that is, interest on the investment and its repayment over a specified number of years related to its lifetime. The sum of these is usually taken as a fixed percentage of investment each year; for example, for a twenty years amortization and 8% interest rate, the annual cost is 0.10185 of the investment.
> Operating costs are primarily for power requirement for

> pumping water and moving air in the system, summed over the yearly operating time of the system. Maintenance costs include repairs, replacement of glass in collectors, or any other costs of keeping the system in operating condition. Consideration of these costs leads to the conclusion that maintenance must be minimized if solar heating is to be economically viable, particularly when labour costs are to be charged as part of the maintenance expense."

Similarly, the cost of auxiliary also consists of the equipment cost (e.g. furnace), the fuel cost,etc...

There are other factors that can further complicate the matter, for example, insurance or real estate taxes or tax relief on residential use of solar energy. For lack of data, and for simplicity, the cost analysis of a solar heating system, which is based on [3], and [4] can be broken down as follows (Canadian dollar is used):

-The cost of the heat pump is \$3000. The annual operating cost cost of the heat pump is:

$$C_1=Q_{hp}xC_{el}$$

where,

Q_{hp}=Amount of electricity consumed by heat pump [KWhr]

C_{el}=Utility energy rate [\$/KWhr]

-The collector cost:

$$C_2=A_cxC_{cl}$$

where,

A_c= Effective collector area [m²]

C_{cl} = Collector cost [\$/m²]

-The storage cost:

$$C_3=C_{st}\,x M_s$$

C_{st} = Storage cost [\$/Kg]

M_s = Storage mass [Kg]

-The cost of pumps, valves, pipes, heat exchanger

$$C_4=200.+10xA_c$$

-The controller cost:

$$C_5=150.$$

-The capital cost of the electric heater (used as auxiliary heater) is $200.

-The cost of auxiliary energy used:

$$C_6=Q_{aux}\,xC_{el}$$

If I is the capital cost of the system, for given amortized period of N years, and a given annual interest of i%, the amortized capital cost in year j is[4]:

$$iI(1+i)^N/[(1.+i)^N-1.]$$

For an annual real inflation rate of fuel at r%, the operating cost in year j is[4]:

$$(Q_{aux}+Q_{hp})C_{el}(1.+r)^j$$

So, the total cost of the system in year j is:

$$C_{sj}=(3000.+A_c xC_{cl}+C_{st}M_s+200.+10xA_c+150.+200.)xi(1+i)^N/[(1+i)^N-1]+(Q_{aux}+Q_{hp})C_{el}(1+r)^j \quad (4.2\text{-}1)$$

The total cummulative cost of the solar system is:

$$C_s=\sum_{j=1}^{N}C_{sj}\ .$$

Assuming that the mortgage life ,N, is equal to the system age, the annual interest rate is 12%, and the annual real inflation rate of electricity and natural gas is 10%. Using the values in [3],we have:

C_{cl}=\$60/m²,

C_{st}=\$0.132/Kg,

C_{el}=\$0.00335/KWhr.(Edmonton Power price)

The assumed conventional system uses natural gas (which is widely available in Alberta) to heat the house. Suppose the cost of the furnace is \$1000, which is also financed in the same amortized period and at the same interest rate as that of the solar system.

Similarly, the total cost of the conventional system at year j is:

$$C_{cvj}=1000.xi(1+i)^N/[(1+i)^N-1]+Q_{cv}C_g\,(1+r)^j$$

The price of natural gas in Edmonton, C_g , is \$2.94/GJ (Northwestern Utilities price, tax included).

4.2.1 The viability of the solar system.

A computer program is written to compare the costs associated respectively with the above two systems when the solar system age varies from 10 to 30 years. With the collector area of 20 m², and the storage mass per unit collector area of 75 Kg, the results are calculated and plotted in Fig.4.8, it is noted that from this figure that the solar heating system is not economically viable. If the collector area is increased to 40 m², the solar heating system is not viable either, as seen in Fig.4.9. Hence, the economic viability of solar heating is possible only if the solar industry is developed enough to decrease the investment cost on solar equipments.

4.2.2 Costs versus mass storage and collector.

Using the same amortized period, annual interest and real inflation rate, the system cummulative costs change with respect to respective variation of the collector area and the mass storage is plotted in Fig.4.10 and Fig.4.11. These figures show that the collector and the mass storage are dominating factors in the total system costs. Therefore, we will not be able to base our design criterions on the energy-saving aspect alone, unless at some future time the capital costs spent on the collector and the mass storage are down.

4.2.3 Costs versus Heat Pump Capacity

Based upon data provided by AAF Ltd. (5718 103 St. Edmonton, Alberta), the heat pump price per kW is approximated by (Fig.4.12):

$$P_{hp} = 250(W_{max}) + 900 \qquad (4.2.3\text{-}1)$$

where P_{hp} =price of heat pump [\$C],

W_{max} =maximum electrical input to heat pump

[kW]

Replacing 3000. in equation (4.2-1) by P_{hp} , and with the same assumption regarding interest rate, amortized period, etc., the cummulative system costs are calculated with various values of W_{max} and plotted in Fig.4.14 (collector area=20 m²) and Fig.4.13 (collector area=100 m²). Like collector and mass storage, the heat pump cost also plays a dominant role in the system cost .

4.3 Recommendation.

The preceding discussion dealt with the thermal and economic aspects of a solar heating system. Based on these results, a few conclusions can be drawn. However, it should be noted here that due to the unavailability of suitable data(for example the only complete set of weather data available to us is from October 1, 1967 to April 1, 1968), the previous results as well as the following comments should be taken with caution.

-The tilt slope of the collector should be made as close as possible to the lattitude.

-Single-glazed collector should be used.

-The building loop pump capacity should be increased to the neighborhood of a proper value if this does not lead to an increase in total cost. Here, for lack of specific pump cost, it is not possible to investigate the economic

-The heat exchanger coefficient should be kept as close to 1. as practically and economically possible. The economic aspect of the system response due to variation of the heat exchanger is not studied for lack of data.

-The heat loss coefficient of the tanks can cause a 2% decline in the saving percentage for a change from 5 to 12KJ/hr C. This points out the need for a better insulated tank though the saving is not much.

-The increased storage mass, heat pump capacity, and collector area respectively help boost the saving percentage but also make the system more costly.

-The use of solar energy in Edmonton is not viable.

In fact, there are many other ignored factors that can further complicate the matter and enhance or destroy the economic aspect of the system. They are such as tax incentive, maintenance costs, property tax,etc...and they are beyond the scope of this thesis.

4.4 Further research.

As mentioned earlier (section 2.4.2 and 4.1.8), the suboptimal controller is not "smart" enough to take in to account the existence condition of the steady-state optimal tank temperatures. However, we believe that there is the possibility to upgrade the thermal performance of the system if further work is done on this area.

Thus far we assume that the process involved is deterministic in nature. In reality, the weather forecast data, which are only probable values are subject to random effects and it is essential that these effects are taken into account in our calculations. The stochastic optimum theory may be useful in studying the stochastic nature of the process.

Another problem which remains unsolved is the implementation of the suboptimal controller on a microprocessor. The calculation of the controller values (equation (2.4.2-1)) is relatively simple. However, if we want to avoid developing arithmetic subroutines (i.e. addition, subtraction, multiplication, etc.), a number-oriented microprocessor (calculator chip) can be used as in [7]. According to T.B.Kent et al[8], as far as technical problems to design a microprocessor controller are concerned they are non-existent. In other words, the implementation of this suboptimal controller is certainly possible. The only problems left are to develop proper hardware and software to support it.

Since utility companies might offer a lower price on electricity produced during off-peak hours, it would be economically advantageous to use the auxiliary electrical energy for the system during that period while reducing peak hour demands of electricity. The study of this problem can also be extended to examine the impact of the system electrical load on the power network.

The above are but a few related areas that needs more studies.

4.5 Conclusion.

Using the KUHN-TUCKER theorem, we have obtained the expression of the steady-state optimal values of the high-side temperature, the low-side temperature, and the heat-pump controller. As weather conditions change, the optimal heat pump controller sequence is numerically evaluated. Based upon this result and upon the results of a process of trial and error, a suboptimal adaptive preview controller is proposed. Its simulation result is analysed and compared with that of the WATSUN approach. Savings of 31% or more are recorded for the suboptimal controller. The thermal and economic behaviour of the system are also investigated to identify areas for possible improvement of the system performance. Further research is also proposed.

Table 4.1

	saving percentage	
Collector area	100 m^2	20 m^2
Zero-glazed	36.61%	30.80%
Single-glazed	42.83%	30.86%
Double-glazed	42.84%	30.72%

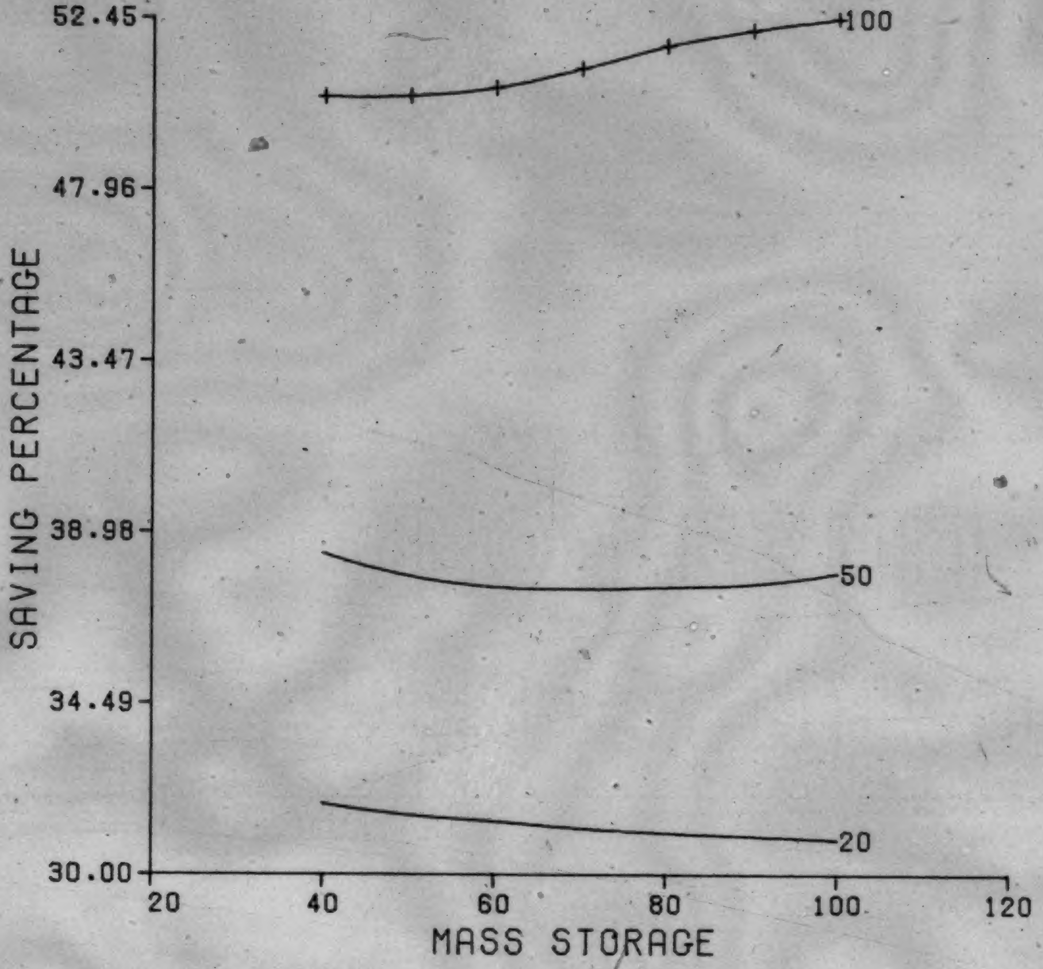

Fig 4.1: Saving percentage versus mass storage

(collector area=100,50,20 m²)

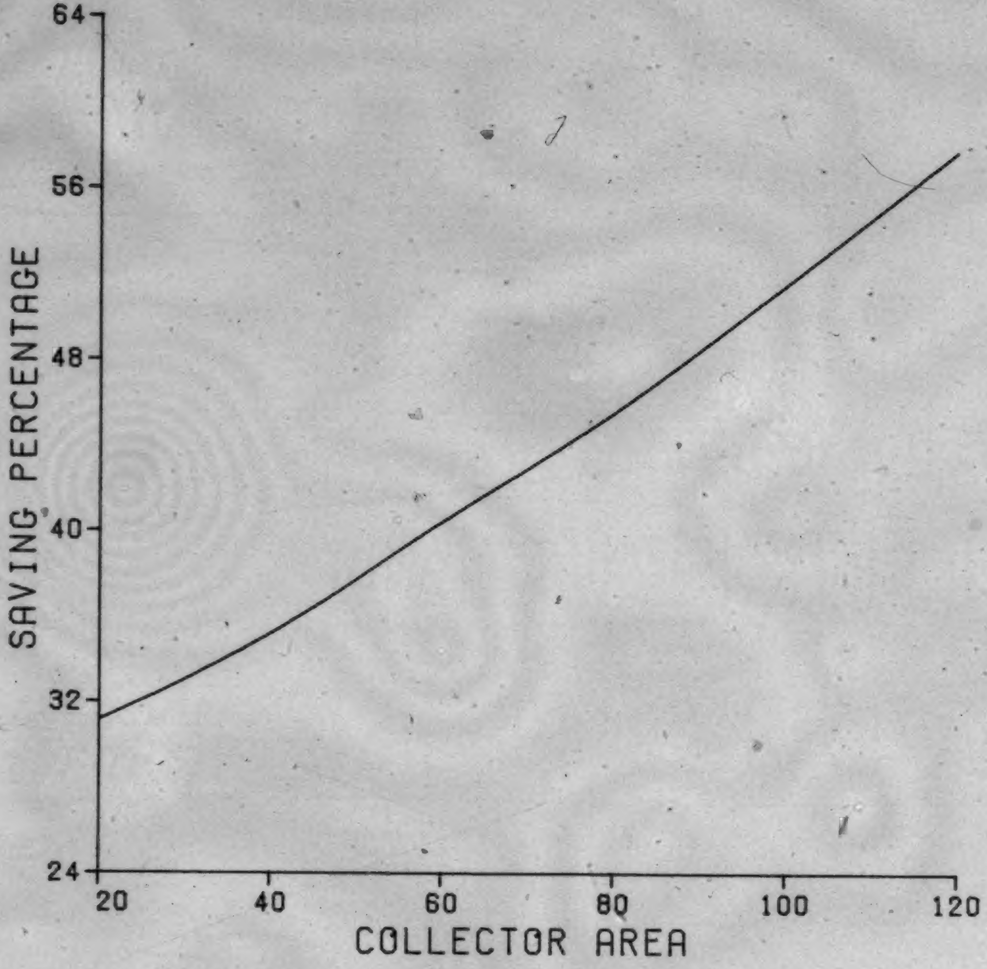

Fig 4.2: Saving percentage versus collector area

(collector area= 20-120 m², mass storage= 75 kG/m²)

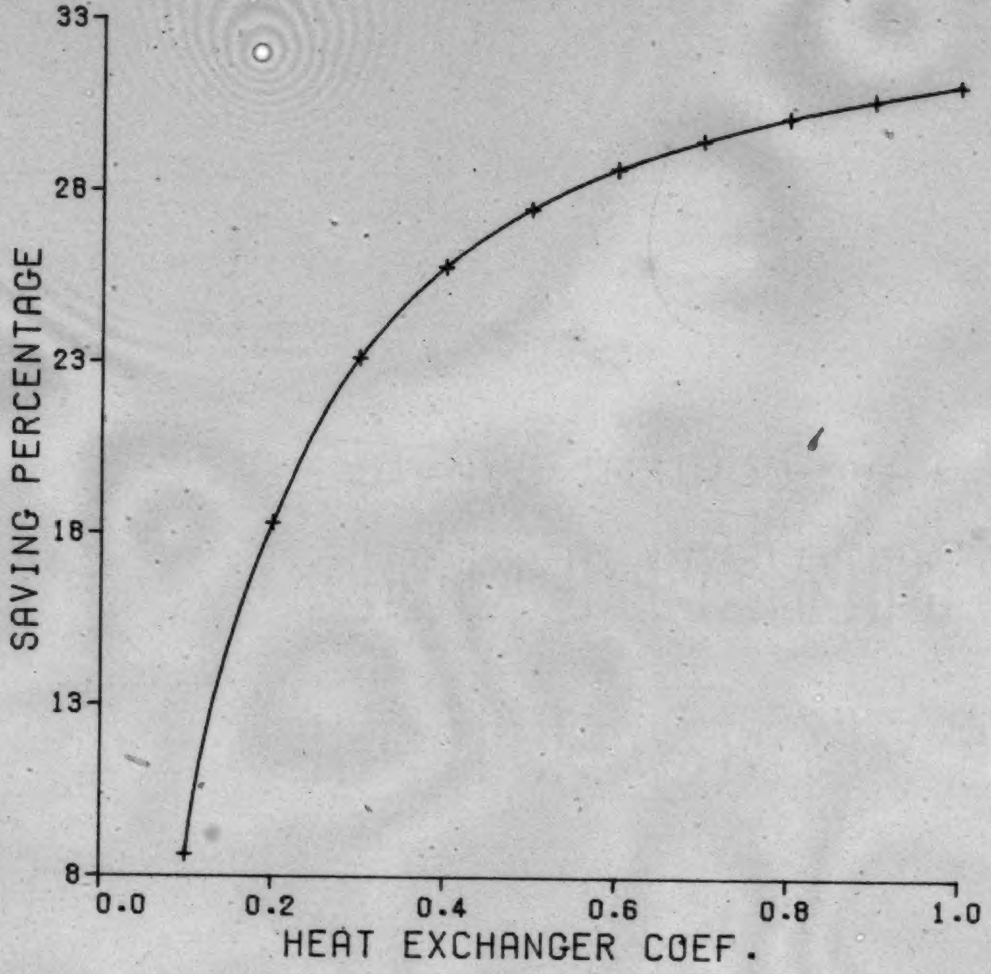

Fig 4.3: Saving percentage versus heat exchanger coefficient

(collector area=20 m²)

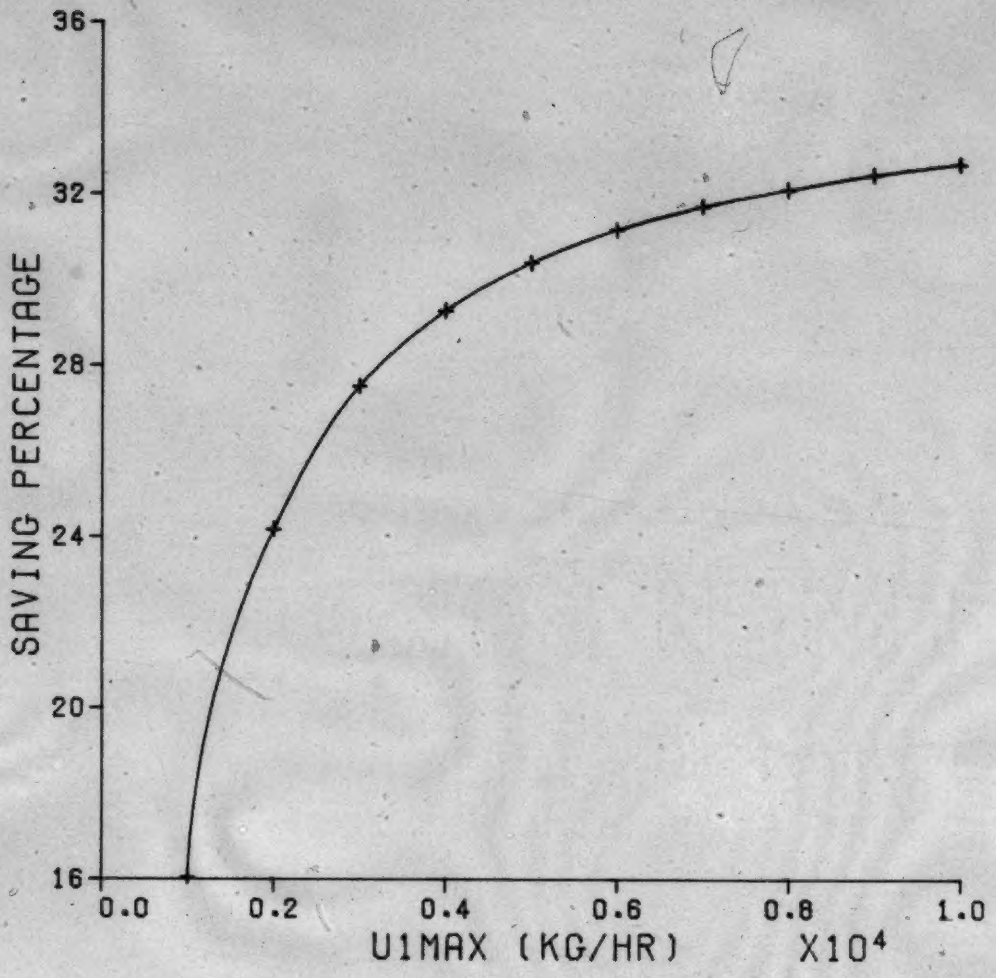

Fig 4.4: Saving percentage versus building loop pump capacity
(collector area=20 m²)

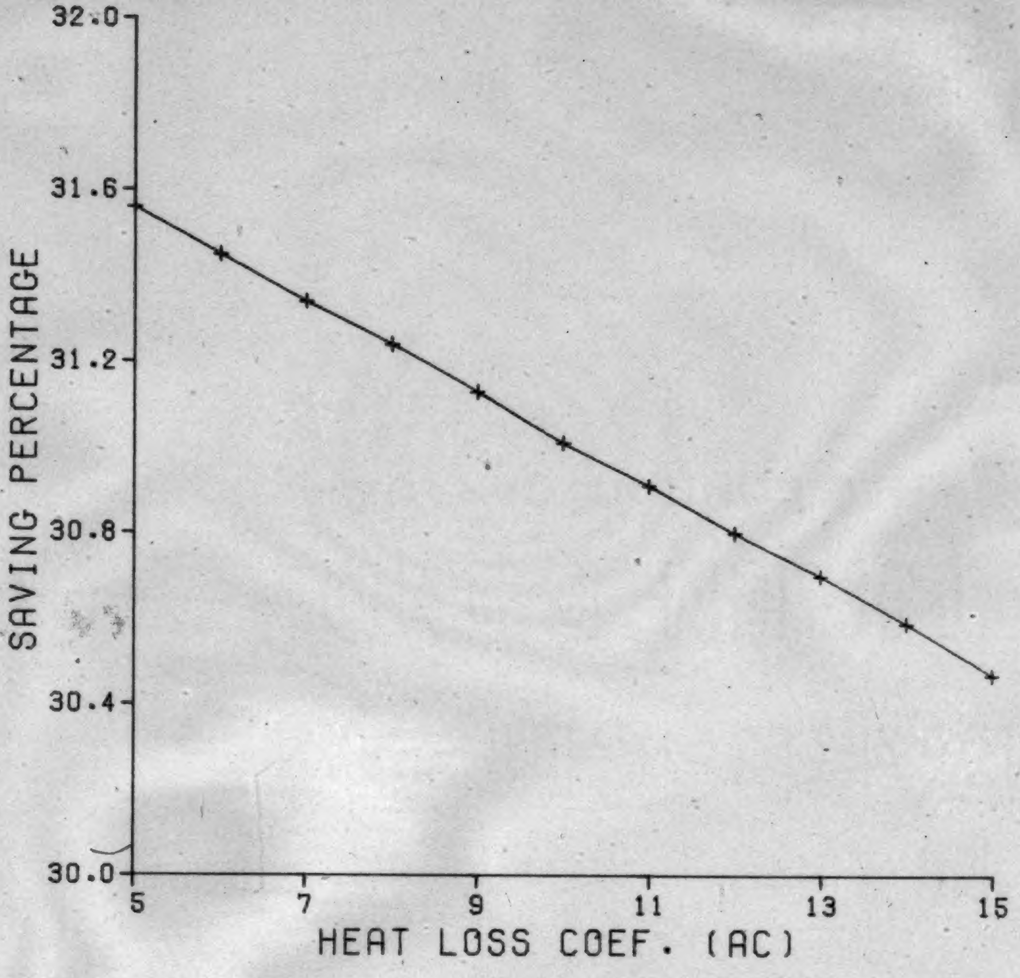

Fig 4.5: Saving percentage versus tanks heat loss coefficient(a_h,a_c) (collector area=20 m²).

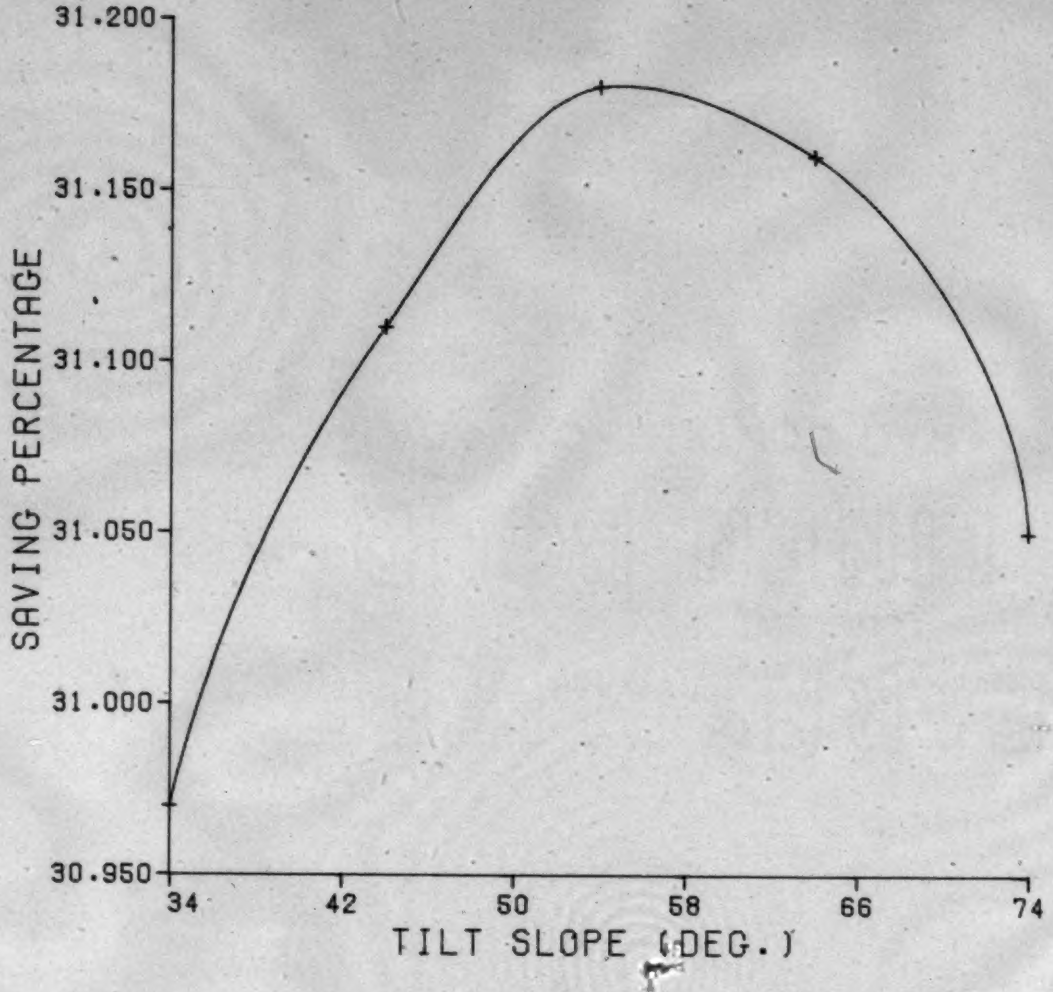

Fig 4.6: Saving percentage versus tilt slope (collector area=20 m²)

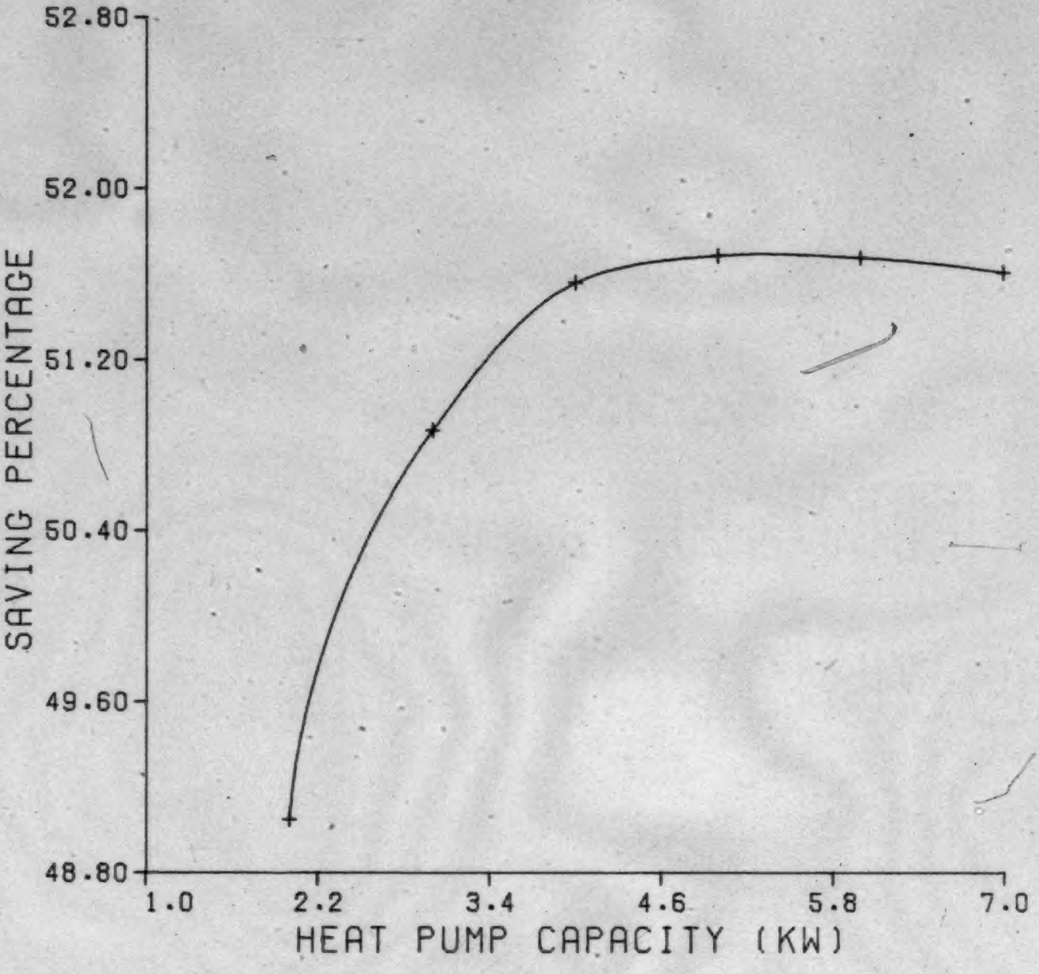

Fig 4.7: Saving percentage versus heat pump capacity

(collector area= 100 m²)

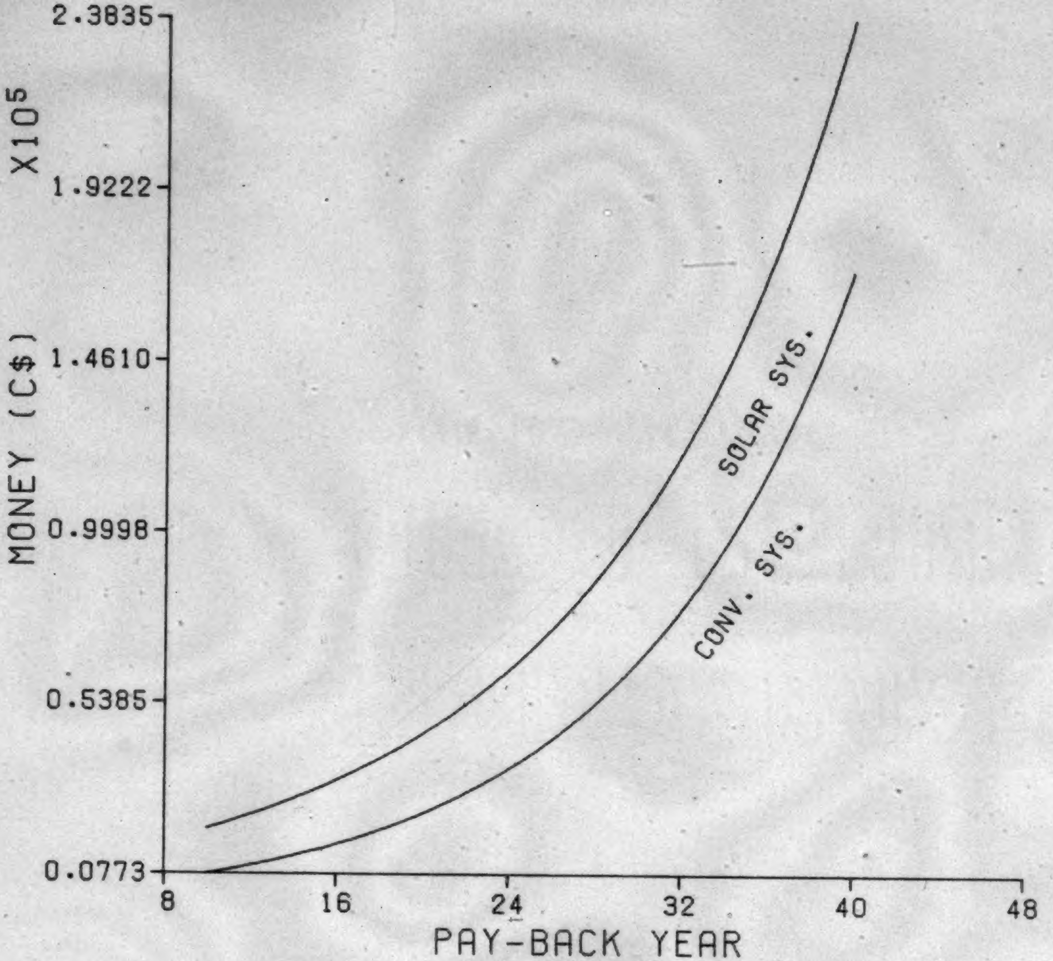

Fig.4.8: Cummulative costs of the solar and conventional systems.

(collector area=20 m²)

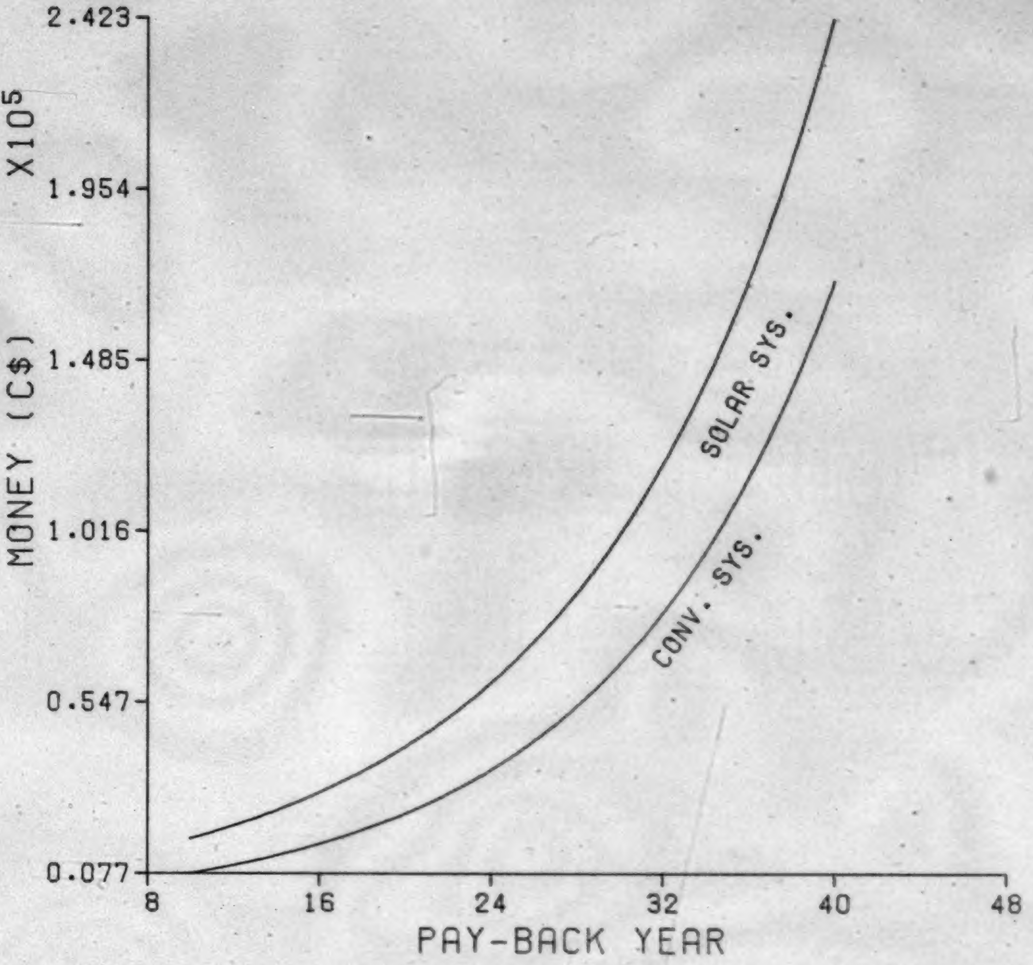

Fig.4.9: Cummulative costs of the solar and the conventional systems.

(collector area=40 m²)

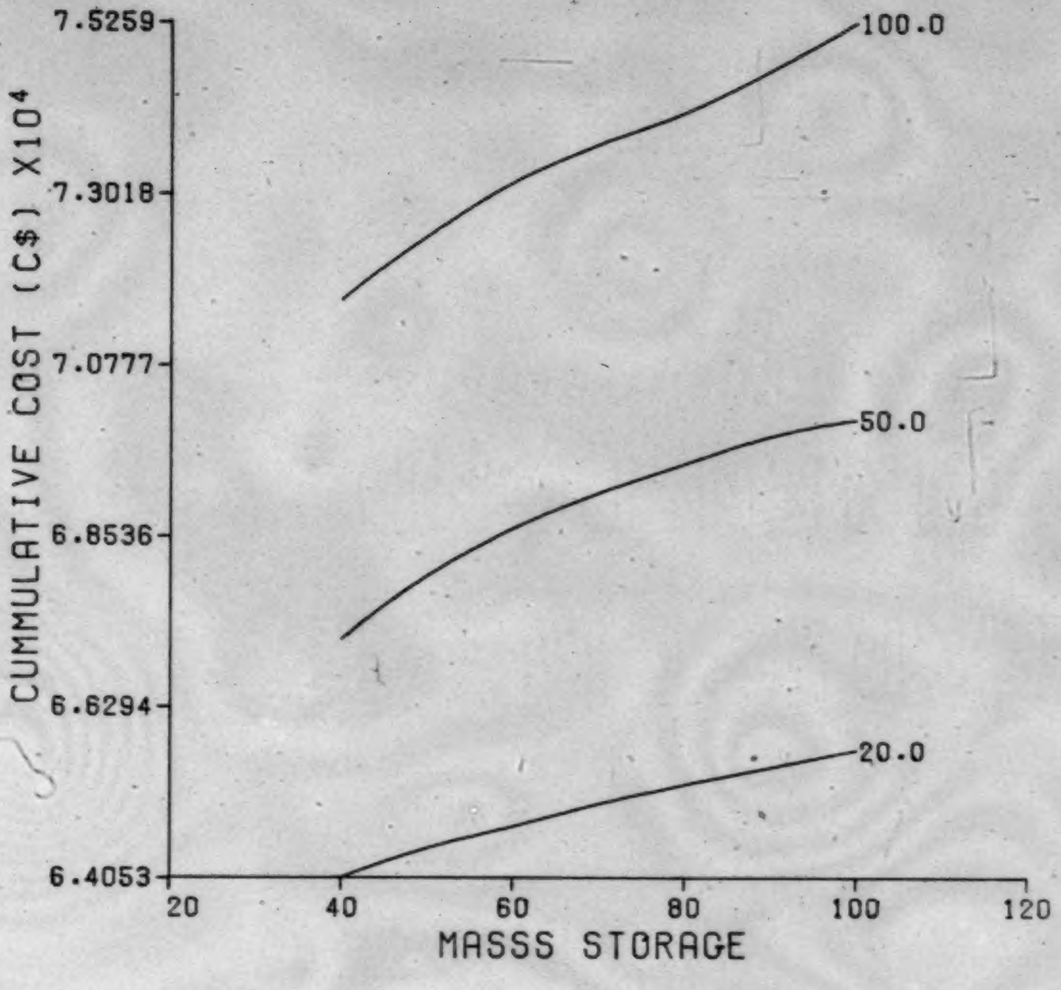

Fig 4.10: Cost versus mass storage(Kg/m²).

(collector area=100,50,20 m²)

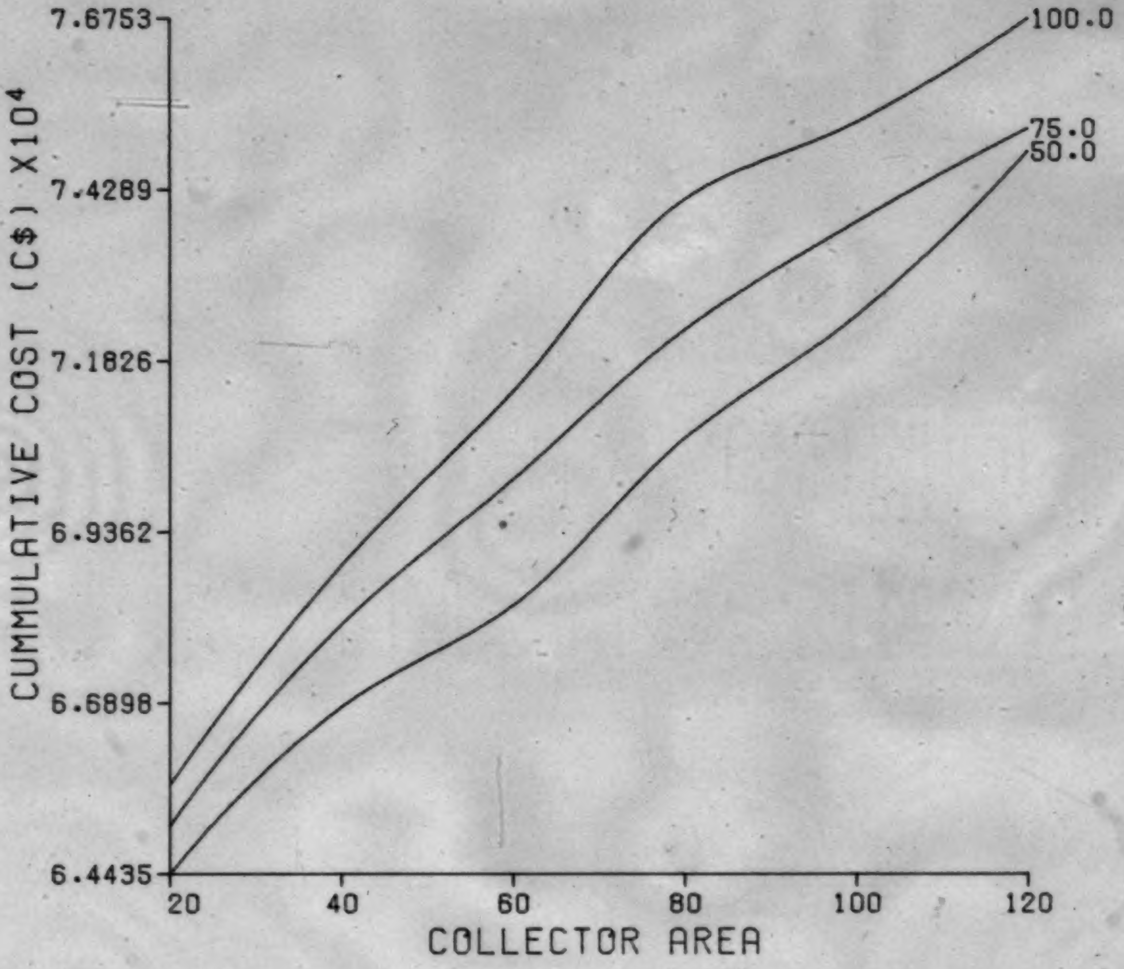

Fig 4.11:Cost versus collector area.

(mass storage=100,75,50 kg/m²)

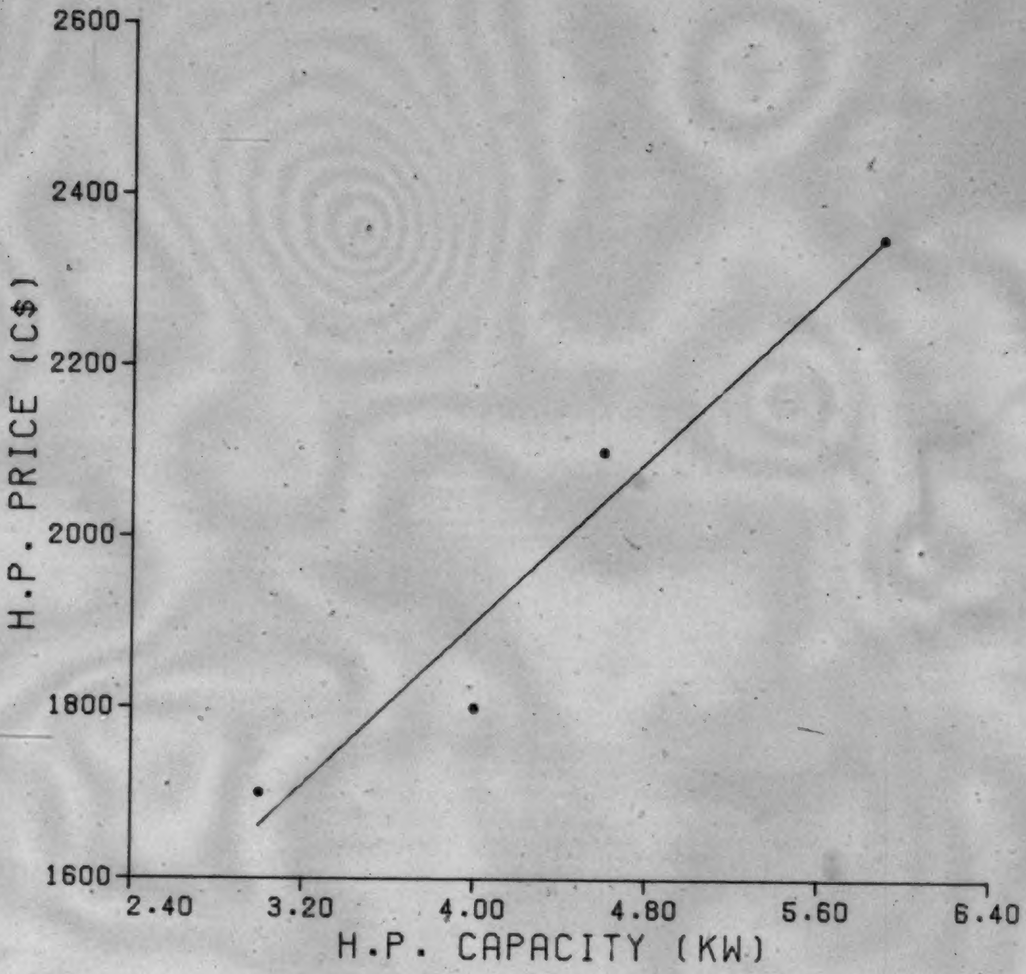

Fig.4.12: Heat pump price versus capacity

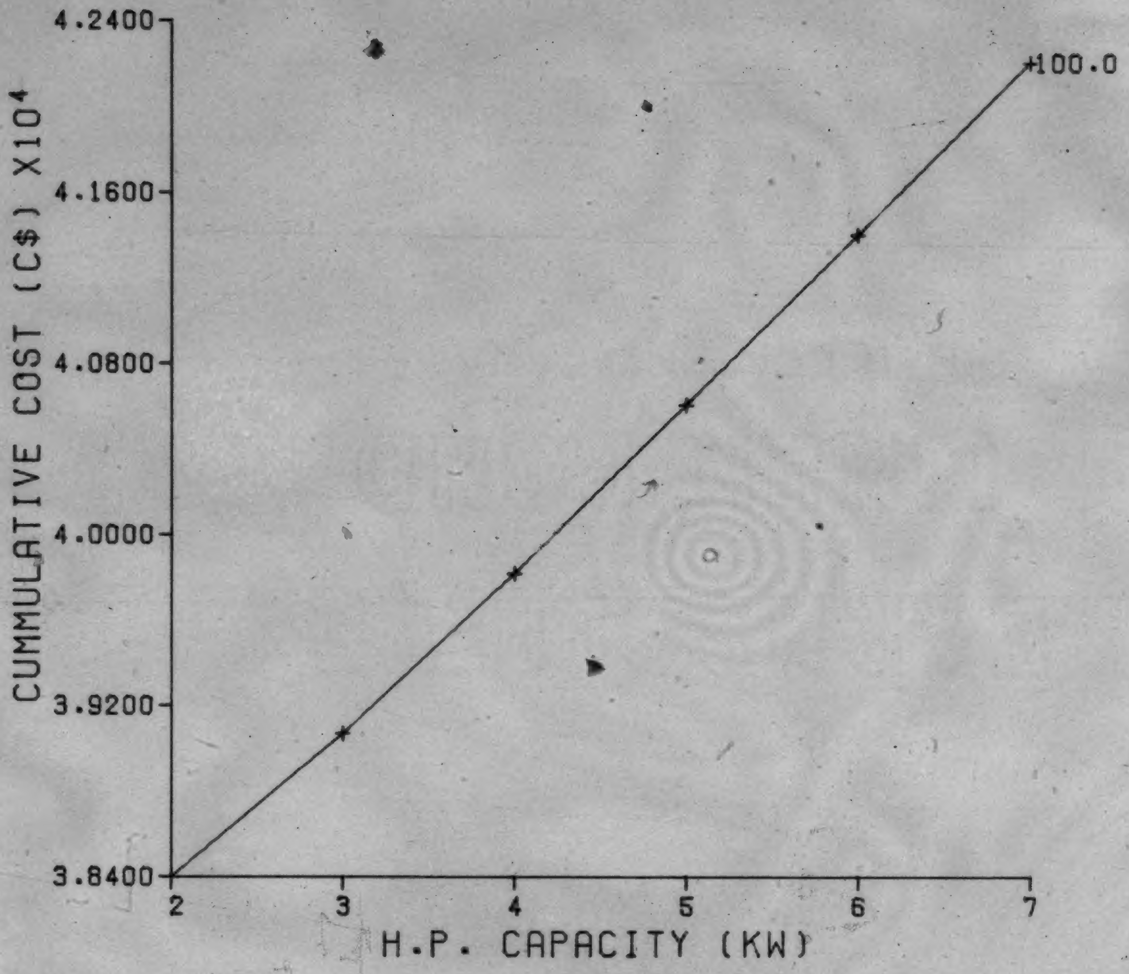

Fig.4.13: cummlative cost versus heat pump capacities

(collector area=100 m²)

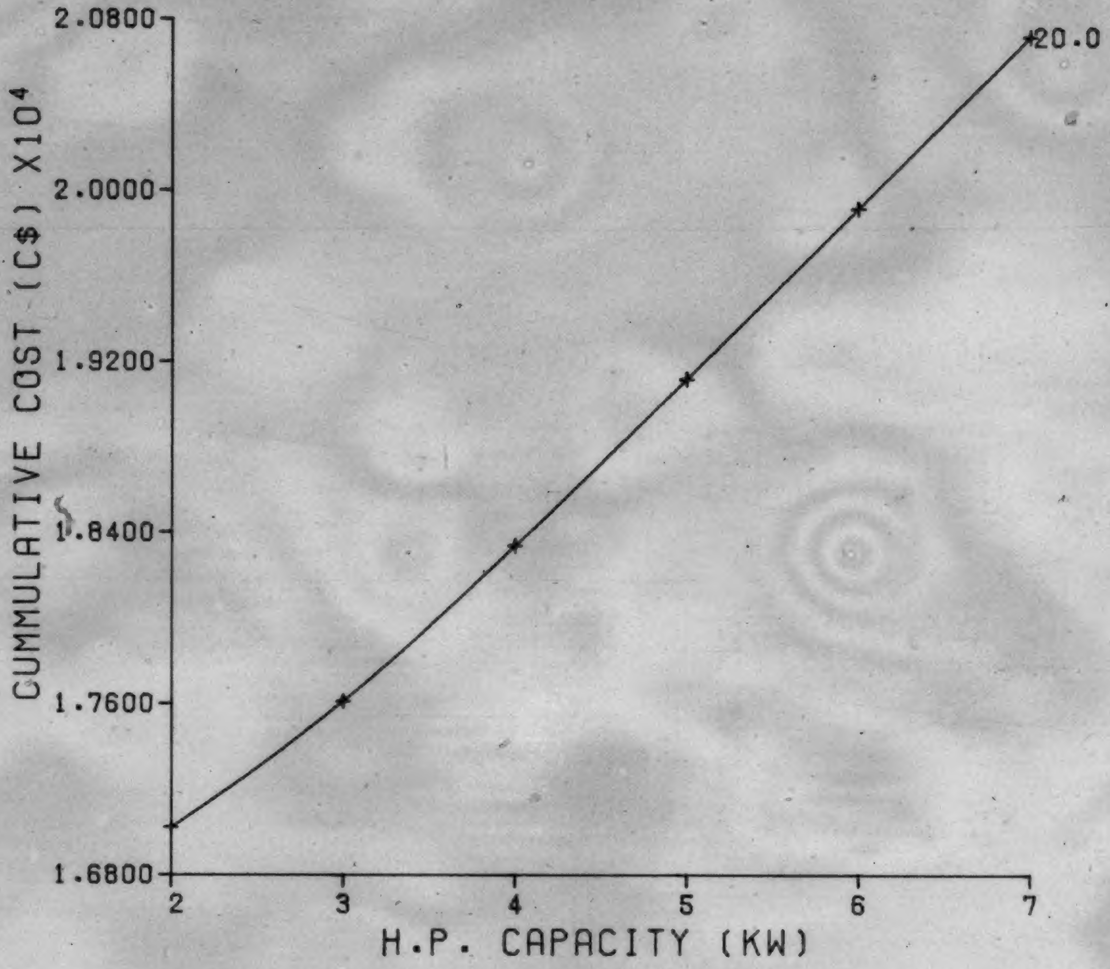

Fig.4.14: Cummulative cost versus heat pump capacities

(collector area=20 m²)

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APPENDIX I

Meteorological data processing

The hourly measurements of the radiation and the ambient temperature are needed for using the suboptimal controller. It is the radiation incident on the tilted collector that is needed for calculating the collected energy (and the gain of the controller), but the only values provided by Environment Canada are the total radiation (H) incident on a horizontal surface. A program based upon the following algorithm was written to process these data[1].

First, calculate the beam (H_b) and the diffused(H_d) components of H:

$$H=H_d+H_b$$

and,

$$H_d=KxH$$

where, K can be calculated by using a correlation:

$$K=0.177 \quad \text{if} \quad K_T\geq 0.75$$

$$K=1.56-1.84K_T \quad \text{if} \quad 0.35\leq K_T<0.75$$

$$K=1.-0.249K_T \quad \text{if} \quad 0.\leq K_T<0.35$$

and $K_T=H/H_0$

with H_0 being the instantaneous extraterrestrial radiation on a horizontal surface:

$H_0=Sc[1+0.033\cos(2\pi N/365)]x[\cos\omega\cos\varphi\cos\delta+\sin\varphi\sin\delta]$

where, Sc= solar constant

N= day of the year

δ= solar declination

φ= lattitude

ω= hour angle

The H_T can be determined from:

$$H_T=R_bH_b+(1+\cos S)\times H_d/2.+\rho(1-\cos S)H/2.$$

S= tilted angle of collector

ρ= ground reflectance factor

R_b is the ratio of beam radiation on a tilted surface to that on a horizontal surface; and if the collector is facing towards the equator, R_b is calculated as follows:

$$R_b=[\cos(\varphi-S)\cos\delta\cos\omega+\sin(\varphi-S)\sin\delta]/[\cos\varphi\cos\delta\cos\omega+\sin\varphi\sin\delta]$$

REFERENCES of APPENDIX I

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APPENDIX II

Program to find the optimal controller sequence

The following program is originally form [1], with weather data and system parameters being purely hypothetical [2].

```
C     PROGRAM TO OPTIMIZE CONTROL FOR
C     HEAT PUMP SYSTEM
      DIMENSION U1(51),U2(51),TH(51),TL(51),
     1   RL(51),COST(51),PCOST(51),GRA(51),
     2   U2L(51),COSTL(51),DIR(51),DI(51),
     3   QAUX(51),GRAD(51),RH(51)
C.........................................
C               PART ONE
C     ESTABLISHMENT OF THE INITIAL TRAJECTORY
C
C.........................................
      DT=0.15
      TB=20.
      TA=-20.
      TH(1)=40.
      TL(1)=5.
      S=1.0
      ITER=1
      J=0
      COST(1)=0.
      TCMIN=1000.
      DO 100 I=1,50
      IF(I.GT.15)S=0.
      IF(I.GT.15)TA=-20.
      IF(I.GT.35)S=1.0
      IF(I.GT.35)TA=-20.
      DELT=TB-TA
      U1(I)=0.5*DELT/(TH(I)-TB)
      IF(U1(I).GT.1)U1(I)=1.
      QAUX(I)=0.1*DELT-0.2*(TH(I)-TB)*U1(I)
      UNUM=0.2*U1(I)*(TH(I)-TB)+0.02*(TH(I)-TA)
      UDEN=20*(1.-.016*(TH(I)-TL(I)))
      U2(I)=UNUM/UDEN
      IF(U2(I).GT.1)U2(I)=1.
      IF((TH(I)-TL(I)).GT.45)U2(I)=0.
      U2L(I)=U2(I)
      RH(I)=UDEN*U2(I)-UNUM
      TERM1=20.*U2(I)*(.715-.016*(TH(I)-TL(I)))
     +      +0.02*(TL(I)-TA)
      TERM2=10.*(S-0.025*(TL(I)-TA))
      IF(TERM2.LT.0)TERM2=0.
      RL(I)=TERM2-TERM1
      TH(I+1)=TH(I)+DT*RH(I)
      TL(I+1)=TL(I)+DT*RL(I)
      COST(I+1)=COST(I)+QAUX(I)+5.7*U2(I)
100   CONTINUE
      TCOST=COST(51)
      WRITE(6,1)ITER,J,TCOST
      WRITE(6,4)
      WRITE(6,2)(U2(I),I=1,50)
      WRITE(6,6)
      WRITE(6,3)(TH(I),I=1,50)
      WRITE(6,8)
      WRITE(6,3)(TL(I),I=1,50)
C.....................................
C                PART TWO
C          EVALUATION OF THE GRADIENTS
C     AND DOING ONE-DIMENSIONED SEARCH
C.....................................
      GMA2=1.
      GM2=1.
125   JP=35
      GA1=GA2
      DMAG2=0.
      GMAG2=0.
150   PCOST(JP)=COST(JP)
      U2(JP)=U2L(JP)+0.01
      DO 200 I=JP,50
      IF(I.GT.15)S=0.
      IF(I.GT.15)TA=-20.
      IF(I.GT.35)S=1.0
      IF(I.GT.35)TA=-20.
      DELT=TB-TA
      U1(I)=0.5*DELT/(TH(I)-TB)
      IF(U1(I).GT.1)U1(I)=1.
      QAUX(I)=0.1*DELT-0.2*(TH(I)-TB)*U1(I)
      UNUM=0.2*U1(I)*(TH(I)-TB)+0.02*(TH(I)-TA)
      UDEN=20*(1.-.016*(TH(I)-TL(I)))
      IF(I.LE.35)GO TO 175
      U2(I)=(1.+1.*(40.-TH(I)))*UNUM/UDEN
175   CONTINUE
      IF(I.EQ.JP)GO TO 180
      IF(U2(I).GT.1)U2(I)=1.
180   IF((TH(I)-TL(I)).GT.45)U2(I)=0.
      IF(U2(I).LT.0)U2(I)=0.
      RH(I)=UDEN*U2(I)-UNUM
      TERM1=20.*U2(I)*(.715-.016*(TH(I)-TL(I)))
     +      +0.02*(TL(I)-TA)
      TERM2=10.*(S-0.025*(TL(I)-TA))
      IF(TERM2.LT.0)TERM2=0.
      RL(I)=TERM2-TERM1
      TH(I+1)=TH(I)+DT*RH(I)
      TL(I+1)=TL(I)+DT*RL(I)
      PCOST(I+1)=PCOST(I)+QAUX(I)+5.7*U2(I)
200   CONTINUE
      GRA(JP)=GRAD(JP)
      GRAD(JP)=(PCOST(51)-TCOST)/0.01
      GMAG2=GMAG2+(GRAD(JP))**2
      DIR(JP)=-0.5*GRAD(JP)+0.5*DI(JP)
      DMAG2=DMAG2+(DIR(JP))**2
      DI(JP)=DIR(JP)
      IF(JP.EQ.1)GO TO 250
      JP=JP-1
      GO TO 150
250   CONTINUE
      GM2=GMA2
      GMA2=GMAG2
      R=GMA2/GM2
      JMIN=1
      DO 600 J=1,10
      G=0.01*J
270   DO 500 I=1,50
      U2(I)=U2L(I)+G*DIR(I)/(DMAG2)**0.5
      IF(I.GT.15)S=0.
      IF(I.GT.15)TA=-20.
      IF(I.GT.35)S=1.0
      IF(I.GT.35)TA=-20.
      DELT=TB-TA
      U1(I)=0.5*DELT/(TH(I)-TB)
      IF(U1(I).GT.1)U1(I)=1.
      QAUX(I)=0.1*DELT-0.2*(TH(I)-TB)*U1(I)
      UNUM=0.2*U1(I)*(TH(I)-TB)+0.02*(TH(I)-TA)
      UDEN=20*(1.-.016*(TH(I)-TL(I)))
      IF(I.LE.35)GO TO 275
      U2(I)=(1.+1.*(40.-TH(I)))*UNUM/UDEN
275   CONTINUE
      IF(U2(I).GT.1)U2(I)=1.
      IF((TH(I)-TL(I)).GT.45)U2(I)=0.
      IF(U2(I).LT.0)U2(I)=0.
      RH(I)=UDEN*U2(I)-UNUM
      TERM1=20.*U2(I)*(.715-.016*(TH(I)-TL(I)))
     +       +0.02*(TL(I)-TA)
      TERM2=10.*(S-0.025*(TL(I)-TA))
      IF(TERM2.LT.0)TERM2=0.
      RL(I)=TERM2-TERM1
      TH(I+1)=TH(I)+DT*RH(I)
      TL(I+1)=TL(I)+DT*RL(I)
      COST(I+1)=COST(I)+QAUX(I)+5.7*U2(I)
500   CONTINUE
      TCOST=COST(51)
      IF(KEEP.EQ.1)GO TO 610
      IF(TCOST.LT.TCMIN)JMIN=J
      IF(TCOST.LT.TCMIN)TCMIN=TCOST
600   CONTINUE
      G=0.01*JMIN
      KEEP=1
      GO TO 270
610   ITER=ITER+1
      DO 620 K=1,30
620   U2L(K)=U2(K)
      KEY=ITER/50
      KEEP=0
      IF((KEY-LKEY).LT.1)GO TO 125
      LKEY=KEY
      WRITE(6,1)ITER,JMIN,TCOST,G
```

```
1     FORMAT(////1X,'ITER=',I4,5X,I4,'COST=',F6.2,F12.8)
      WRITE(6,4)
4     FORMAT(//1X,'U2=')
      WRITE(6,2)(U2(I),I=1,50)
2     FORMAT(1X,10F8.2)
      WRITE(6,5)
5     FORMAT(//1X,'GRAD=')
      WRITE(6,3)(GRAD(I),I=1,30)
3     FORMAT(1X,10F8.2)
      WRITE(6,6)
6     FORMAT(//1X,'TH=')
      WRITE(6,3)(TH(I),I=1,50)
      WRITE(6,8)
      WRITE(6,3)(TL(I),I=1,50)
8     FORMAT(//1X,'TL=')
C.......................................
C           PART THREE
C      CONDITION TO STOP
C.......................................
      IF(ABS(GM2-GMA2)/GM2.LE.0.1)GO TO 125
      STOP
      END
```

REFERENCES of APPENDIX II

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2.R.E.Rink and H.Q.Le, "Multivariable Feedback Control of Bilinear Processes in HVAC systems", Proceedings of the 24th MidWest Symposium on Circuits and Systems, Alburquerque, NM., pp.777-781, (1981).

APPENDIX III

Program to simmulate the WATSUN approach and the suboptimal controller.

```
C.....................................................
C THIS PROGRAM IS USED TO SIMMULATE THE
C SUBOPTIMAL CONTROLLER AND THE WATSUN APPROACH
C.....................................................
C
C
C.....................................................
C TO RUN THE PROGRAM:
C   *USE ADAPTIVE CONTROLLER:
C    CALL READIN,WEATHR,AVERA,OPTSET,SYSANA
C   *USE WATSUN APPROACH:
C    CALL READIN,WEATHR,SYSWAT
C......................................................
      COMMON /INITL/THI,TLI,U2I,NDAYS
      COMMON /IFILE/IAMB,IAVER,ICOST,IDATA,IOPTI,
     +               IPARA,IRAD
      COMMON /WEATH/SLOPE,AZMTH,XLAT,RHO
      COMMON /PARA/AB,AC,AAC,AH,ALPTAU,CC,CH,COPMAX,U1MAX,
     + U2MAX,UL,TB,TMAX,ZETA,DT,FU3
      CALL READIN
      CALL WEATHR
      CALL AVERA
 10   CALL OPTSET
      CALL SYSANA
C     CALL SYSWAT
      STOP
      END
C.................................
      BLOCK DATA
      COMMON /PARA/AB,AC,AAC,AH,ALPTAU,CC,CH,COPMAX,U1MAX,
     + U2MAX,UL,TB,TMAX,ZETA,DT,FU3
      COMMON /INITL/THI,TLI,U2I,NDAYS
      COMMON /IFILE/IAMB,IAVER,ICOST,IDATA,IOPTI,
     +              IPARA,IRAD
      COMMON /WEATH/SLOPE,AZMTH,XLAT,RHO
      DATA IAMB,IAVER,ICOST,IDATA,IOPTI,IPARA,
     +     IRAD/2,3,7,9,4,8,1/
      END


C......................................................
C THIS SUBROUTINE USED TO DETERMINE THE PURCHASED ENERGY
C CONSUMED BY THE SYSTEM, USING THE SUBOPTIMAL CONTROLLER.
C......................................................
C
      SUBROUTINE SYSANA
      DIMENSION TH(200),TL(200)
      REAL MASS,MONEY,INSOL
      COMMON /PARA/AB,AC,AAC,AH,ALPTAU,CC,CH,COPMAX,
     + U1MAX,U2MAX,UL,TB,TMAX,ZETA,DT,FU3
      COMMON /INITL/THI,TLI,U2I,NDAYS
      COMMON /IFILE/IAMB,IAVER,ICOST,IDATA,IOPTI,
     +              IPARA,IRAD
      COMMON /WEATH/SLOPE,AZMTH,XLAT,RHO
C....................................................
 10   FORMAT(10X,2E15.8)
 11   FORMAT(36X,E15.8)
C120  FORMAT(/,1H ,'D',I4,'COST= ',E15.8,' COP=',F6.2,
C    + /,' REF. COST=',E15.8,' %SAVE=',F6.2,/)
 110  FORMAT(1H ,'S=',E10.4,' TA=',F6.2,' TH=',F6.2,
     + ' TL=',F6.2,' U2= ',F5.2)
C160  FORMAT(/,1H ,'ANNUAL COST = ',E15.8,' ENERGY COST=',
C    + E10.5,/,' COL. AREA=',F6.1,' MASS STORAGE=',E10.5)
C....................................................
      REWIND ICOST
      COLMAS=75.
      ISAMP=4
 5    CONTINUE
C........................................................
C SCALE FACTOR IS 1000
C........................................................
      AREA=AAC*1000.
      MASS=(CC*1000.)*2./4.18
      REWIND IDATA
      REWIND IOPTI
```

```
C.......................
C   INITIAL CONDITONS
C.......................
      IDAY=1
      TH(1)=THI
      TL(1)=TLI
      U2=U2I
      TOTAL=0.
      RTOTAL=0.
      SOLAR=0.
      INSOL=0.
C......................................
C SOLAR=SOLAR COLLECTION
C INSOL=INSOLATION
C HPOUT=OUTPUT OF HEAT-PUMP
C HPIN=INPUT OF HEAT-PUMP
C HPOUTD=OUTPUT OF HEAT-PUMP IN A DAY
C HPIND=INPUT OF HEAT-PUMP IN A DAY
C......................................
      HPOUTD=0.
      HPIND=0.
      HPOUT=0.
      HPIN=0.
      IMARK=0
      KOUNT=1
 90   CONTINUE
C     STOTAL=0.
```

```
C     DO 200 ITER=1,10
      COST=0.
      RCOST=0.
      DO 150 I=2,24
      TH(I)=0.
      TL(I)=0.
  150 CONTINUE
      THN=0.
C.....................
      READ(IOPTI,11)THN
      DO 100 I=1,24
      READ(IDATA,10)TA,S
C...................................................
C
C     IF(IDAY.EQ.NDAYS)GO TO 29
C     IF(IDAY.NE.KOUNT)GO TO 30
C 29  WRITE(ICOST,110)S,TA,TH(I),TL(I),U2
   30 IF(IMARK.NE.1)GO TO 40
      CALL ONETAN(S,TA,TH(I),TL(I),TH(I+1),
     +     TL(I+1),COST1,RCOST1,SOLAR1,ISAMP)
      U2=0.
      HP=0.
      GO TO 50
C...................................................
C
  40  CALL TEMP(S,TA,TH(I),TL(I),U2,MARK,
     + TH(I+1),TL(I+1),THN,COST1,HP,RCOST1,SOLAR1,ISAMP)
```

```
 50   COST=COST+COST1
      SOLAR=SOLAR+SOLAR1
      INSOL=INSOL+S
      RCOST=RCOST+RCOST1
      HPOUTD=HPOUTD+HP
      HPIND=HPIND+U2*U2MAX
C     WRITE(ICOST,110)S,TA,TH(I),TL(I),U2
      HPOUT=HPOUT+HP
      HPIN=HPIN+U2*U2MAX
      S=0.
      TA=0.
      IF(TL(I+1).NE.TH(I+1))GO TO 179
      IF(TL(I+1).GT.24.)GO TO 100
      IMARK=0
      GO TO 100
 179  IF(TL(I+1).LT.(TH(I+1)+5.))GO TO 100
      AT=(TL(I+1)+TH(I+1))/2.
      TL(I+1)=AT
      TH(I+1)=AT
      IMARK=1
 100  CONTINUE
C     IF(IDAY.EQ.NDAYS)GO TO 129
C     IF(IDAY.NE.KOUNT)GO TO 130
      IF(HPIND.NE.0.)COPD=HPOUTD/HPIND
      IF(HPIND.EQ.0.)COPD=20.
      HPOUTD=0.
      HPIND=0.
      SAVE=(RCOST-COST)*100./RCOST
C129  WRITE(ICOST,119)IDAY,COST
 119  FORMAT(/,1H ,'D',I4,'COST= ',E15.8,/)
C     WRITE(ICOST,120)IDAY,COST,COPD,RCOST,SAVE
C.......................
 130  TH(1)=TH(25)
      TL(1)=TL(25)
 180  IDAY=IDAY+1
      TOTAL=TOTAL+COST
C     STOTAL=STOTAL+COST
      RTOTAL=RTOTAL+RCOST
      IF(IDAY.GT.NDAYS)GO TO 300
 200  CONTINUE
C     KOUNT=KOUNT+10
C     WRITE(ICOST,400)STOTAL
 400  FORMAT(/,1H ,'COST OF 10 DAYS=',E15.8)
      IF(IDAY.LE.NDAYS)GO TO 90
 300  CONTINUE
C     WRITE(ICOST,400)STOTAL
C     WRITE(ICOST,159)TOTAL
 159  FORMAT(/,1H ,'SEASONAL COST (KJ)=',E15.8)
      COPHP=HPOUT/HPIN
 161  FORMAT(1H ,'TOTAL ENERGY=',E10.5,'COL. AREA=',E10.5,
     +'MASS=',E10.5,/,' COP=',F4.2,' REF. COST=',E10.5,
     + ' SYSCOP=',F6.4,' COLCOP=',F6.2,/,' UL=',F6.2,
     + ' ALPTAU=',F4.2,' U1MAX=',F8.2,' AC=',F4.1,/,
     + ' U2MAX=',F6.2,'KW',' ZETA=',
     + F4.1,' SLOPE=',F6.2,' DEG',' COLMAS=',
     + F6.2,/,' NUMBER OF SAMPLES=',I3,' PER HOUR')
      SYSCOP=(RTOTAL-TOTAL)/RTOTAL
      COLCOP=SOLAR/INSOL
      ACCOEF=AC*1000.
      UU1MAX=U1MAX*1000.
      UU2MAX=U2MAX*1000./3600.
      SSLOPE=SLOPE*180./3.1415927
      WRITE(ICOST,161)TOTAL,AREA,MASS,COPHP,RTOTAL,SYSCOP,
     +  COLCOP,UL,ALPTAU,UU1MAX,ACCOEF,UU2MAX,ZETA,SSLOPE,
     +  COLMAS,ISAMP
      RETURN
      END


      SUBROUTINE TEMP(S,TA,TH1,TL1,U2,MARK,
     + TH2,TL2,THN,COST1,HP,RCOST1,SOLAR1,ISAMP)
      COMMON /PARA/AB,AC,AAC,AH,ALPTAU,CC,CH,COPMAX,U1MAX,
     + U2MAX,UL,TB,TMAX,ZETA,DT,FU3
      THF1=TH1
      TLF1=TL1
      DT=1./FLOAT(ISAMP)
      TERM1=0.
      COST1=0.
      HP=0.
      RCOST1=0.
      SOLAR1=0.
      ITER=1
      DELT=TB-TA
 20   TL11=TL1
 100   IF(TH1.LE.TB)U1=0.
       IF(TH1.LE.TB)GO TO 111
      U1=AB*DELT/(U1MAX*(TH1-TB)*ZETA)
  111 IF(U1.GT.1.)U1=1.
      IF(U1.LE.0.)U1=0.
      QAUX=AB*(TB-TA)-U1*U1MAX*(TH1-TB)*ZETA
      IF(QAUX.LE.0.)QAUX=0.
      UNUM=U1*U1MAX*ZETA*(TH1-TB)+AC*(TH1-TA)
C.............................
 120  CONTINUE
C.............................
C..........................
C IF TL1 .LT. TA USE AMBIENT AS
C SOURCE FOR HEAT PUMP.
C............................
      IF(TL1.LT.TA)TL11=TA
      UDEN=U2MAX*(1.+2.5*(1.-(TH1-TL11)/TMAX))
      TERM2=AAC*FU3*(S*ALPTAU-UL*(TL1-TA))
      IF(TERM2.LE.0.)TERM2=0.
      IF(TL1.GE.80.)TERM2=0.
      U2=0.5*U2+0.5*UNUM/UDEN*(1.+(THN-TH1))
 101  CONTINUE
      COP=1.+(COPMAX-1.)*(1.-(TH1-TL1)/45.)
      IF(COP.LT.(1.+AH/(ZETA*U1MAX)))U2=0.
      IF(U2.GT.1.)U2=1.
      IF(U2.LE.0.)U2=0.
      RH=UDEN*U2-UNUM
      IF(TL1.LT.TA)TERM1=AC*(TL1-TA)+TERM1
      IF(TL1.LT.TA)GO TO 107
      TERM1=U2MAX*U2*2.5*(1.-(TH1-TL11)/TMAX)+
     + AC*(TL1-TA)+TERM1
C.............................
C IF TL1 IS LESS THAN TA, HEAT PUMP WON'T
C EXTRACT HEAT FROM LOW-TEMPERATURE TANK.
C...........................................
 107  RL=TERM2-TERM1
      TH2=TH1+DT*RH/CH
      TL2=TL1+DT*RL/CC
      COST1=(QAUX+U2MAX*U2)*DT+COST1
      HP=U2*UDEN*DT+HP
      RCOST1=AB*(TB-TA)*DT+RCOST1
      SOLAR1=TERM2/AAC*DT+SOLAR1
      TL1=TL2
      TH1=TH2
      ITER=ITER+1
      IF(ITER.LE.ISAMP)GO TO 20
      TH1=THF1
      TL1=TLF1
      RETURN
      END
```

```
C THIS IS THE SUBROUTINE TO READ
C PARAMETER DATA OF THE SYSTEM.
      SUBROUTINE READIN
      COMMON /PARA/AB,AC,AAC,AH,ALPTAU,CC,
     + CH,COPMAX,U1MAX,U2MAX,UL,TB,TMAX,ZETA,DT,FU3
      COMMON /INITL/THI,TLI,U2I,NDAYS
      COMMON /IFILE/IAMB,IAVER,ICOST,IDATA,IOPTI,
     +              IPARA,IRAD
      COMMON /WEATH/SLOPE,AZMTH,XLAT,RHO
 10   FORMAT(//,3E10.3)
 20   FORMAT(/,8E10.3)
 30   FORMAT(/,4E10.3)
 40   FORMAT(/,3E10.3)
 50   FORMAT(/,E10.3)
 60   FORMAT(/,3E10.3,I5)
C
C.............................
      REWIND IPARA
C..............................
C      STORAGE DATA
C..............................
      READ(IPARA,10)AC,AH,U1MAX
C
C..............................
C      COLLECTOR DATA
C..............................
```

```
      READ(IPARA,20)AAC,ALPTAU,FU3,UL,SLOPE,XLAT,
     + AZMTH ,RHO
C
C................................
C        BUILDING DATA
C................................
      READ(IPARA,30)AB,CB,TB,ZETA
C
C................................
C        HEAT-PUMP DATA
C..................................
      READ(IPARA,40)COPMAX,TMAX,U2MAX
C
C.................................
C        SAMPLING TIME (STEP TIME)
      READ(IPARA,50)DT
C
C...........................
C        INITIAL CONDITIONS
C...........................
      READ(IPARA,60)THI,TLI,U2I,NDAYS
C
C...............................
C        SCALED DATA(TO MINIMIZE ROUND-OFF ERROR)
C................................................
C
C IN THE FOLLOWING , THE STORAGE IS ASSUMED TO BE
```

```
C 75KG PER COLLECTOR SQUARE METER
C........................................
      CC=4.18*AAC*75.
      CH=CC
C........................................
      AC=AC/1000.
      AH=AH/1000.
      U1MAX=U1MAX/1000.
      AAC=AAC/1000.
      AB=AB/1000.
      U2MAX=U2MAX*3600./1000.
      CC=CC/1000.
      CH=CH/1000.
      RETURN
      END



C........................................
C SUBROUTINE OPTSET USED TO DETERMINE
C ALL OPTIMUM SETS OF TH,TL BASED UPON
C THE AVERAGE VALUE OF AMBIENT TEMPERATURE
C AND RADIATION.
C..........................................
C
      SUBROUTINE OPTSET
      COMMON /PARA/AB,AC,AAC,AH,ALPTAU,CC,CH,COPMAX,U1MAX,
     + U2MAX,UL,TB,TMAX,ZETA,DT,FU3
      COMMON /IFILE/IAMB,IAVER,ICOST,IDATA,IOPTI,
     +               IPARA,IRAD
      COMMON /INITL/THI,TLI,U2I,NDAYS
C
 10   FORMAT(7X,2E15.8)
 15   FORMAT(1H ,'S= ',E15.8,' TA= ',F6.2,' TH0= ',E15.8)
C
C...............................
      REWIND IAVER
      REWIND IOPTI
C...............................
      DO 20 I=1,NDAYS
      READ(IAVER,10)TA,S
      TH0=TB+AB*(TB-TA)/(U1MAX*ZETA)
      WRITE(IOPTI,15)S,TA,TH0
      S=0.
      TA=0.
 20   CONTINUE
      RETURN
      END


C***********************************************
      SUBROUTINE WEATHR
C     ......TO PROCESS METEOROLOGICAL FILES
```

```
C**************************************************
      INTEGER RADTN(24),IT(24)
      COMMON /IFILE/IAMB,IAVER,ICOST,IDATA,IOPTI,
     +              IPARA,IRAD
      COMMON /WEATH/SLOPE,AZMTH,XLAT,RHO
      COMMON /INITL/THI,TLI,U2I,NDAYS
 22   FORMAT(17X,24(I6,1X))
 25   FORMAT(17X,24(I6,1X))
 21   FORMAT(15X,I2)
 31   FORMAT(3X,'D',I3,'H',I2,2E15.8)
 100  FORMAT(30X,40X)
      SINSLP=SIN(SLOPE)
      COSSLP=COS(SLOPE)
      SINAZM=SIN(AZMTH)
      COSAZM=COS(AZMTH)
      SINLAT=SIN(XLAT)
      COSLAT=COS(XLAT)
      TERM1=SINLAT*COSSLP-COSLAT*SINSLP*COSAZM
      TERM2=COSLAT*COSSLP+SINLAT*SINSLP*COSAZM
      TERM3=SINSLP*SINAZM
      RD1=(1.+COSSLP)/4.
      RD2=(1.-COSSLP)/2.
C
      REWIND IAMB
      REWIND IRAD
      REWIND IDATA
C TO FORWARD ONE RECORD,COMMENT, OF IRAD AND
```

```
C IAMB DATA FILES
      READ(IAMB,100)
      READ(IRAD,100)
C ......
      DO 11 IDAY=1,NDAYS
C..... 30 SHOULD BE CHANGED TO 365 OR 366 (DAYS)
         DAY=IDAY+273
C..... OCTOBER 1ST IS THE 274TH DAY OF THE YEAR
         IF(DAY.GT.365.)DAY=DAY-365.
  41     READ(IAMB,21)KCRIT
         IF(KCRIT.EQ.78)GO TO 42
         KCRIT=0
         GO TO 41
  42     BACKSPACE IAMB
         READ(IAMB,22)IT
 43     READ(IRAD,21)KCRIT
         IF(KCRIT.EQ.61)GO TO 44
         GO TO 43
  44     BACKSPACE IRAD
         READ(IRAD,22)RADTN
          RHO=RHO*RD2
          SC=4871.0*(1.+0.33*COS(1.72142E-02* DAY))
          DECL=0.40928*SIN(1.72142E-02*(284.+DAY))
          SINDEC=SIN(DECL)
          COSDEC=COS(DECL)
          TERM4=SINDEC*TERM1
          TERM5=COSDEC*TERM2
         TERM6=COSDEC*TERM3
         TERM7=SINDEC*SINLAT
         TERM8=COSDEC*COSLAT
C.......
      DO 10 IHR=1,24
        TA=IT(IHR)*0.1
        H=RADTN(IHR)
C.......
        IF(H.LE.0.)GO TO 8
        HRANG=0.2618*(11.5-IHR)
         SINHR=SIN(HRANG)
         COSHR=COS(HRANG)
C........
         COSTT=TERM4+COSHR*TERM5+SINHR*TERM6
           IF(COSTT.LE.0.)GO TO 8
C........
         COSTZ=TERM7+COSHR*TERM8
           IF(COSTZ.LE.0.)GO TO 8
C.........
         RB=COSTT/COSTZ
           IF((RB.LE.0.).OR.(RB.GT.5.))RB=0.
         HEX=SC*COSTZ
           IF(HEX.LT.H)HEX=H
         XKT=H/HEX
C.........
         IF(XKT.LT.0.35)GO TO 5
         IF(XKT.LT.0.75)GO TO 6
```

```
            HD=H*0.1769
            GO TO 7
   5        HD=H*(1.0-0.248857*XKT)
            GO TO 7
   6        HD=H*(1.55699-1.84013*XKT)
   7       CONTINUE
C.........
            HB=H-HD
C           HBT=(0.5*HD+HB)*RB
C           HDT=HD*RD1+H*RHO
C           HT=HBT+HDT
            HT=RB*HB+2.*RD1*HD+RHO*RD2*H
              GO TO 9
   8        HT=0.
   9         WRITE(IDATA,31)IDAY,IHR,TA,HT
   10        CONTINUE
   11       CONTINUE
        END FILE IDATA
       RETURN
           END




C..............................
C THIS PROGRAM USED TO DETERMINE THE
C AVERAGE VALUES OF AMBIENT TEMPERATURE
C AND RADIATION.
```

```
C...............................
      SUBROUTINE AVERA
      DIMENSION T(24),S(24),TAV(200),SAV(200)
      COMMON /IFILE/IAMB,IAVER,ICOST,IDATA,IOPTI,
     +              IPARA,IRAD
      COMMON /INITL/THI,TLI,U2I,NDAYS
C.............................
      REWIND IDATA
      REWIND IAVER
C.............................
      DO 10 I=1,NDAYS
      SAV(I)=0.
       TAV(I)=0.
      DO 11 J=1,24
      READ(IDATA,20)T(J),S(J)
 20   FORMAT(10X,2E15.8)
      TAV(I)=TAV(I)+T(J)
      SAV(I)=SAV(I)+S(J)
      T(J)=0.
      S(J)=0.
 11   CONTINUE
      TAV(I)=TAV(I)/24.
      SAV(I)=SAV(I)/24.
      WRITE(IAVER,30)I,TAV(I),SAV(I)
 30   FORMAT(3X,'D',I3,2E15.8)
 10   CONTINUE
      RETURN
      END



C THIS PROGRAM USED TO SIMMULATE THE
C APPROACH ADOPTED BY WATSUN PROGRAM.
C HERE HEAT PUMP IS USED IN ON-OFF
C MODE ONLY.
      SUBROUTINE SYSWAT
      DIMENSION TH(200),TL(200)
      COMMON /PARA/AB,AC,AAC,AH,ALPTAU,CC,CH,COPMAX,U1MAX,
     + U2MAX,UL,TB,TMAX,ZETA,DT,FU3
      COMMON /INITL/THI,TLI,U2I,NDAYS
      COMMON /IFILE/IAMB,IAVER,ICOST,IDATA,IOPTI,
     +              IPARA,IRAD
      WRITE(ICOST,121)
C.................................
      REWIND IDATA
C.................................
C.............................
C    INITIAL CONDITIONS
C.............................
      ISAMP=4
      IDAY=1
      TH(1)=THI
      TL(1)=TLI
      HPOUT=0.
      HPIN=0.
      HPIND=0.
      HPOUTD=0.
      TOTAL=0.
      KOUNT=1
   90 CONTINUE
      STOTAL=0.
      DO 300 ITER=1,10
      COST=0.
      DO 150 IT=2,25
      TH(IT)=0.
  150 TL(IT)=0.
      DO 100 I=1,24
      READ(IDATA,10)TA,S
   10 FORMAT(10X,2E15.8)
C     IF(IDAY.EQ.NDAYS)GO TO 29
C     IF(IDAY.NE.KOUNT)GO TO 30
C29   WRITE(ICOST,110)S,TA,TH(I),TL(I),U2
   30 CALL TEMP1(S,TA,TH(I),TL(I),
     +TH(I+1),TL(I+1),THN,COST1,HP,U2,ISAMP)
      S=0.
      TA=0.
      COST=COST+COST1
      HPOUT=HPOUT+HP*CC/1000.
      HPIN=HPIN+U2*U2MAX*CC/1000.
      HPIND=HPIND+U2*U2MAX*CC/1000.
      HPOUTD=HPOUTD+HP*CC/1000.
```

```
 100  CONTINUE
C.......................
 121  FORMAT(//,1H ,10X,'WATSUN APPROACH',/)
C     IF(IDAY.EQ.NDAYS)GO TO 129
C     IF(IDAY.NE.KOUNT)GO TO 130
      IF(HPIND.NE.0.)COPD=HPOUTD/HPIND
      IF(HPIND.EQ.0.)COPD=20.
      HPOUTD=0.
      HPIND=0.
C129  WRITE(ICOST,119)IDAY,COST
 119  FORMAT(/,1H ,'D',I4,'COST= ',E15.8)
C129  WRITE(ICOST,120)IDAY,COST,COPD
C.......................
 130  TH(1)=TH(25)
      TL(1)=TL(25)
      IDAY=IDAY+1
      TOTAL=TOTAL+COST
      STOTAL=STOTAL+COST
      IF(IDAY.GT.NDAYS)GO TO 310
 300  CONTINUE
C     KOUNT=KOUNT+10
      WRITE(ICOST,400)STOTAL
 400  FORMAT(/,1H ,'COST OF 10 DAYS=',E15.8)
      IF(IDAY.LE.NDAYS)GO TO 90
 310  COP=HPOUT/HPIN
      WRITE(ICOST,400)STOTAL
C     WRITE(ICOST,201)TOTAL
```

```
 201  FORMAT(/,1H ,'SEASONAL COST (KJ)=',E15.8)
      WRITE(ICOST,200)TOTAL,COP
 200  FORMAT(/,1H ,' TOTAL COST= ',E15.8,' COP=',F6.2/)
 120  FORMAT(/,1H ,'D',I3,1X,'COST= ',E15.8,' COP=',F6.2/)
 110  FORMAT(1H ,'S=',E10.4,' TA=',F6.2,' TH=',F6.2,
     + ' TL=',F6.2,' U2=',F6.2)
      RETURN
      END

      SUBROUTINE TEMP1(S,TA,TH1,TL1,
     + TH2,TL2,THN,COST1,HP,U2,ISAMP)
      COMMON /PARA/AB,AC,AAC,AH,ALPTAU,CC,CH,COPMAX,U1MAX,
     + U2MAX,UL,TB,TMAX,ZETA,DT,FU3
      THF1=TH1
      TLF1=TL1
      DT=1./FLOAT(ISAMP)
      COST1=0.
      HP=0.
      TERM1=0.
      DELT=TB-TA
      ITER=1
 20   TL11=TL1
      IF(TL1.LT.20.)GO TO 100
C....................................
C IF LOW-TEMPERATURE TANK COULD PROVIDE
C HEAT TO BUILDING, IT'S WELCOME.
```

```
C.........................................
      U1P=AB*DELT/(U1MAX*(TL1-TB))
      IF(U1P.GT.1.)U1P=1.
      IF(U1P.LT.0.)U1P=0.
      QAUX=AB*(TB-TA)-U1P*U1MAX*(TL1-TB)
      IF(QAUX.LE.0.)QAUX=0.
      IF(QAUX.NE.0.)GO TO 115
C.........................................
C IN CASE LOW-TEMPERATURE TANK CANNOT PROVIDE
C ENOUGH HEAT, HIGH-TEMPERATURE TANK WOULD
C COME IN.
C.........................................
      UNUM=AC*(TH1-TA)
      TERM1=U1P*U1MAX*(TL1-TB)
      GO TO 120
  115 U1=QAUX/(U1MAX*(TH1-TB))
      IF(U1.GE.1.)U1=1.
      IF(U1.LT.0.)U1=0.
      QAUX=QAUX-U1*U1MAX*(TH1-TB)
      IF(QAUX.LE.0.)QAUX=0.
      UNUM=U1*U1MAX*(TH1-TB)+AC*(TH1-TA)
      TERM1=U1P*U1MAX*(TL1-TB)
      GO TO 120
  100 U1=AB*DELT/(U1MAX*(TH1-TB))
      IF(U1.GT.1.)U1=1.
      IF(U1.LT.0.)U1=0.
      QAUX=AB*(TB-TA)-U1*U1MAX*(TH1-TB)
      IF(QAUX.LE.0.)QAUX=0.
      UNUM=U1*U1MAX*(TH1-TB)+AC*(TH1-TA)
 120  CONTINUE
C........................
C IF TL1 .LT. TA USE AMBIENT AS
C SOURCE FOR HEAT PUMP.
C..........................
      IF(TL1.LT.TA)TL11=TA
      UDEN=U2MAX*(1.+2.5*(1.-(TH1-TL11)/TMAX))
      TERM2=AAC*FU3*(S*ALPTAU-UL*(TL1-TA))
      IF(TERM2.LE.0.)TERM2=0.
C..........................
C SOLAR ENERGY DUMPED IF TL1>20.
C...........................
      IF(TL1.GT.20.)TERM2=0.
C.......................................
C OTHERWISE, TURN ON HEAT-PUMP,
C ELECTRICAL INPUT TO THE HEAT PUMP IS
C APPROXIMATED BY THE FOLLOWING EXPRESSION
C........................................
      U2=(2.+(TL1-TH1+44.)/25.)*3.6/U2MAX
      IF(U2.GT.1.)U2=1.
      IF(U2.LT.0.)U2=0.
      RH=UDEN*U2-UNUM
      IF(TL1.LT.TA)TERM1=AC*(TL1-TA)+TERM1
      IF(TL1.LT.TA)GO TO 107
      TERM1=U2MAX*U2*2.5*(1.-(TH1-TL11)/TMAX)+
     + AC*(TL1-TA)+TERM1
C...........................
C IF TL1 IS LESS THAN TA, HEAT PUMP WON'T
C EXTRACT HEAT FROM LOW-TEMPERATURE TANK.
C........................................
 107  RL=TERM2-TERM1
      TH2=TH1+DT*RH/CH
      TL2=TL1+DT*RL/CC
      COST1=(QAUX+U2MAX*U2)*DT+COST1
      HP=U2*UDEN*DT+HP
      TL1=TL2
      TH1=TH2
      ITER=ITER+1
      IF(ITER.LE.ISAMP)GO TO 20
      TH1=THF1
      TL1=TLF1
      RETURN
      END


      SUBROUTINE ONETAN(S,TA,TH1,TL1,TH2,TL2,COST1,RCOST1,
     +                            SOLAR1,ISAMP)
C..................................................
C THIS SUBROUTINE USED IN THE CASE WHERE TWO TANKS ARE
C TO BECOME ONE-TANK SYSTEM,I.E. S HIGH AND TA LOW
C..................................................
```

```
      COMMON /PARA/AB,AC,AAC,AH,ALPTAU,CC,CH,COPMAX,U1MAX,
     + U2MAX,UL,TB,TMAX,ZETA,DT,FU3
      THF=TH1
      DT=1./FLOAT(ISAMP)
      ITER=1
      SOLAR1=0.
      RCOST1=0.
      COST1=0.
      DELT=TB-TA
      TT1=TH1
10    CONTINUE
      U1=AB*DELT/(U1MAX*(TT1-TB))
      IF(U1.GT.1.)U1=1.
      IF(U1.LT.0.)U1=0.
      QAUX=AB*(TB-TA)-U1*U1MAX*(TT1-TB)
      IF(QAUX.LE.0.)QAUX=0.
      UNUM=U1*U1MAX*(TT1-TB)+AH*(TT1-TA)+
     +                          AC*(TT1-TA)
      TERM=AAC*FU3*(S*ALPTAU-UL*(TT1-TA))
      IF(TERM.LE.0.)TERM=0.
      IF(TT1.GE.90.)TERM=0.
      RTT=TERM-UNUM
      TT2=TT1+DT*RTT/(CC+CH)
      COST1=QAUX*DT+COST1
      RCOST1=AB*(TB-TA)*DT+RCOST1
      SOLAR1=TERM/AAC*DT+SOLAR1
      ITER=ITER+1
```

```
TT1=TT2
IF(ITER.LE.ISAMP)GO TO 10
TH1=THF
TL1=THF
TH2=TT2
TL2=TT2
RETURN
END
```

APPENDIX IV

Properties of the system eigenvalues

The dynamic equations for the solar-assisted heat pump system are (equations (2.3-1a) and (2.3-1b):

$$C_h\dot{T}_h = -\xi u_1(T_h-T_b) - a_h(T_h-T_a) + u_2[1 + (COP_{max}-1)(1 - \frac{T_h-T_c}{T_{max}})]$$

$$C_c\dot{T}_c = -u_2[(COP_{max}-1)(1 - \frac{T_h-T_c}{T_{max}})] - a_c(T_c-T_a) +$$
$$A_cF(u_3)[S - U_L(T_c - T_a)]$$

or:

$$C_h\dot{T}_h = [-\xi u_1 - a_h - u_2\frac{(COP_{max}-1)}{T_{max}}]T_h + \frac{u_2(COP_{max}-1)T_c}{T_{max}} +$$
$$\xi u_1T_b + a_hT_a + u_2COP_{max} \qquad (IV.1)$$

$$C_c\dot{T}_c = u_2\frac{(COP_{max}-1)}{T_{max}}\,T_h + [-u_2\frac{(COP_{max}-1)}{T_{max}} - a_c - A_cF(u_3)U_L]T_c$$
$$-u_2(COP_{max}-1) + a_cT_a + A_cF(u_3)(S + U_LT_a) \qquad (IV.2)$$

$$\text{Let}\quad \underline{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} T_h \\ T_c \end{bmatrix}$$

$$\underline{\underline{\bar{\Phi}}} = \begin{bmatrix} \dfrac{-\xi u_1 - a_h - u_2 \dfrac{(COP_{max}-1)}{T_{max}}}{C_h} & \dfrac{u_2(COP_{max}-1)}{T_{max}C_h} \\ \\ \dfrac{u_2(COP_{max}-1)}{T_{max}C_c} & \dfrac{\dfrac{-u_2(COP_{max}-1)}{T_{max}} - a_c - A_c F(u_3)U_L}{C_c} \end{bmatrix}$$

$$\underline{\Gamma} = \begin{bmatrix} \dfrac{\xi u_1 T_b + a_h T_a + u_2 COP_{max}}{C_h} \\ \\ \dfrac{-u_2(COP_{max}-1) + a_c T_a + A_c F(u_3)(S + U_L T_a)}{C_c} \end{bmatrix}$$

Equations (IV.1) and (IV.2) become:

$$\dot{\underline{x}} = \underline{\underline{\bar{\Phi}}}\,\underline{x} + \underline{\Gamma} \tag{IV.3}$$

In the following, we will prove that the eigenvalues e.g. (IV.3) are always nonpositive.

Proof: According to the Gerschgorin theorem [1], if we designate λ_1, λ_2 as the eigenvalues of the system described by (IV.3), we obtain:

$$\left| \lambda_1 - \frac{\left[-\xi u_1 - a_h - u_2 \dfrac{(COP_{max}-1)}{T_{max}}\right]}{C_h} \right| \leq \left| \frac{u_2(COP_{max}-1)}{T_{max}C_h} \right| \tag{IV.4}$$

and,

$$\left| \lambda_2 - \frac{\dfrac{-u_2(COP_{max}-1)}{T_{max}} - a_c - A_cF(u_3)U_L}{C_c} \right| \leq \left| \frac{u_2(COP_{max}-1)}{T_{max}C_c} \right| \quad (IV.5)$$

$$\text{Let } f_1 = \lambda_1 - \left\{ \frac{-\xi u_1 - a_h - u_2\dfrac{(COP_{max}-1)}{T_{max}}}{C_h} \right\} \quad (IV.6)$$

$$f_2 = \lambda_2 - \left\{ \frac{\dfrac{-u_2(COP_{max}-1)}{T_{max}} - a_c - A_cF(u_3)U_L}{C_c} \right\} \quad (IV.7)$$

If $f_1 \geq 0$ since $COP_{max} - 1 > 0$ and $u_2 > 0$, (IV.4) becomes:

$$\lambda_1 - \left\{ \frac{-\xi u_1 - a_h - u_2\dfrac{(COP_{max}-1)}{T_{max}}}{C_h} \right\} \leq \frac{u_2(COP_{max}-1)}{T_{max}C_h}$$

$$\text{or} \quad \lambda_1 \leq -\frac{(\xi u_1 + a_h)}{C_h} \quad (IV.8)$$

Hence, λ_1 is nonpositive.

If $f_1 \leq 0$, (IV.6) becomes:

$$f_1 = \lambda_1 - \left\{ \frac{-\xi u_1 - a_h - u_2 \frac{(COP_{max}-1)}{T_{max}}}{C_g} \right\} \leq 0$$

or

$$\lambda_1 \leq - \left\{ \frac{\xi u_1 + a_h + u_2 \frac{(COP_{max}-1)}{T_{max}}}{C_h} \right\} \qquad \text{(IV.9)}$$

Therefore, λ_1 is always nonpositive.

Similarly, we can also prove that

$$\lambda_2 \leq - \frac{(a_c + A_c F(u_3) U_L)}{C_c} \qquad \text{(IV.10)}$$

or

$$\lambda_2 \leq - \left\{ \frac{a_c + A_c F(u_3) U_L + u_2 \frac{(COP_{max}-1)}{T_{max}}}{C_c} \right\} \qquad \text{(IV.11)}$$

If a_h and a_c are different from zero, from (IV.8), (IV.9), (IV.10), (IV.11), we find that the eigenvalues of the system are always negative no matter what the value of u_1, u_2, $F(u_3)$ are.

Let $u_1 = F(u_3) = 1$, matrix $\bar{\underline{\Phi}}$ reduces to:

$$\bar{\underline{\Phi}} = \underline{A} + \underline{B}\, u_2$$

where $\underline{A}$ and $\underline{B}$ are coefficient matrices of system (3.1-10). Hence, with nonzero values of a_h and a_c, the eigenvalues of the system described by

$$\dot{\underline{x}} = \bar{\underline{\Phi}}\, \underline{x} = (\underline{A} + \underline{B}\, u_2)\, \underline{x}$$

are always negative regardless of the values of u_2.

REFERENCES of APPENDIX IV

1. G.W. Stewart, "Introduction to Matrix Computations", Academic Press, New York, p. 302, (1973).

APPENDIX V

Deadbeat Control of Bilinear Plant

In the following, the controllability and optionality problems of a general discrete-time bilinear system are investigated. Although, the results look promising from the viewpoint of the suboptimality, it is imperative that more work needs be done since there are still unsolved problems with this approach (for example, there are problems of implementing the controller as well as of setting up the state-transition matrix, etc.)

For a continuous bilinear plant, its behaviours can also be examined in terms of this approach, since, as we will show, the discrete-time equivalence of a continuous bilinear plant is also discrete-time bilinear. Assuming that, the general continuous bilinear system equations can be written as follows:

$$\dot{\underline{X}}(t) = (\underline{H} + \sum_{i=1}^{N_{u_i}} u_i \underline{E}_{u_i} + \sum_{i=1}^{N_{v_i}} v_i \underline{E}_{v_i})\underline{x}(t) + \underline{D}_v \underline{u} + \underline{D}_w \underline{w} + \underline{D}_o$$

or $$\dot{\underline{X}}(t) = \underline{F}\,\underline{x}\,(t) + \underline{D}_v\,\underline{v} + \underline{D}_w\,\underline{w} + \underline{D}_o \qquad \text{(V.1)}$$

where

$$\underline{F} = \underline{H} + \sum_{i=1}^{N_{n_i}} u_i \underline{E}_{u_i} + \sum_{i=1}^{N_{v_i}} v_i \underline{E}_{v_i}$$

$\underline{H}$, $\underline{E}_{u_i}$, $\underline{E}_{v_i}$, $\underline{D}_v$, $\underline{D}_w$, $\underline{D}_o$ = constant matrices

$\underline{u}$, $\underline{v}$, $\underline{w}$ = control vectors

The homogeneous solutions to the above differential equation is

$$\underline{x}_h(t) = e^{\underline{F}(t-t_o)}\,\underline{x}(t_o)$$

where, by definitions, the exponential matrix is equal to:

$$e^{\underline{F}(t-t_o)} = \underline{\underline{I}} + \underline{F}(t-t_o) + \underline{F}^2\frac{(t-t_o)^2}{2!} + \ldots$$

or by linear approximation, we have

$$e^{\underline{F}(t-t_o)} \doteq \underline{I} + \underline{F}(t-t_o)$$

Hence,

$$\underline{x}_h(t) = [\underline{I} + \underline{F}(t-t_o)]\;\underline{x}\;(t_o) \qquad (V.2)$$

The particular solution to (V.1) is obtained by using the familiar method ,i.e. variation of parameters. Assuming that, the solution is in the form

$$\underline{x}_p(t) = e^{\underline{F}(t-t_o)}\,\underline{z}(t) \qquad (V.3)$$

Substituting (V.3) into (V.1), we obtain

$$\underline{F}e^{\underline{F}(t-t_o)}\,\underline{z}(t) + e^{\underline{F}(t-t_o)}\,\dot{\underline{z}}(t) = \underline{F}\;\underline{x}_p(t) + \underline{D}_v\;\underline{v} + \underline{D}_w\;\underline{w} + \underline{D}_o$$

or
$$\underline{F}e^{\underline{F}(t-t_o)}\,\underline{z}(t) + e^{\underline{F}(t-t_o)}\,\dot{\underline{z}}(t) = \underline{F}e^{\underline{F}(t-t_o)}\,\underline{z}(t) + \underline{D}_v\;\underline{v} + \underline{D}_w\;\underline{w} + \underline{D}_o$$

This implies

$$e^{\underline{F}(t-t_o)}\,\dot{\underline{z}}(t) = \underline{D}_v\,\underline{v} + \underline{D}_w\,\underline{w} + \underline{D}_o$$

so

$$\dot{z}(t) = e^{-\underline{F}(t-t_o)}\,(\underline{D}_v\,\underline{v} + \underline{D}_w\,\underline{w} + \underline{D}_o)$$

$$\underline{z}(t) = \int_{t_o}^{t} e^{-F(\zeta-t_o)}\,(\underline{D}_v\,\underline{v} + \underline{D}_w\,\underline{w} + \underline{D}_o)\,d\zeta \qquad (V.4a)$$

Suppose, vector $(\underline{D}_v\,\underline{v} + \underline{D}_w\,\underline{w} + \underline{D}_o)$ does not depend on ζ, (V.4a) can be written

$$\underline{z}(t) = \int_{t_o}^{t} e^{\underline{F}(t_o-\zeta)}\,d\zeta\,(\underline{D}_v\,\underline{v} + \underline{D}_w\,\underline{w} + \underline{D}_o) \qquad (V.4b)$$

Thus, from (V.4b) and (V.3)

$$\underline{x}_p(t) = e^{\underline{F}(t-t_o)}\,\underline{z}(t)$$

$$= \int_{t_o}^{t} e^{-F(\zeta-t\ )}\,d\zeta\,(\underline{D}_v\,\underline{v} + \underline{D}_w\,\underline{w} + \underline{D}_o) \qquad (V.5)$$

The complete solution to (V.1) is

$$\underline{x}(t) = \underline{x}_h(t) + \underline{x}_p(t)$$

Since, we wish to use this solution over one sample period, let $t = nT + T$ and $t_o = nT$, we have:

$$\underline{x}(nT + T) = \underline{x}_h(nT + T) + \underline{x}_p(nT + T)$$

$$= (\underline{I} + \underline{F}T)\, \underline{x}(nT) + \int_{nT}^{nT+T} e^{\underline{F}(nT+T-\zeta)} d\zeta\, (\underline{D}_v\, \underline{v} + \underline{D}_w\, \underline{w} + \underline{D}_o) \tag{V.6}$$

In equation (V.6), we assume that (using zero-order hold)

$$\underline{u}(\zeta) = \underline{u}(nT) \quad \text{for} \quad nT \leq \zeta < nT + T$$

$$\underline{v}(\zeta) = \underline{v}(nT) \quad\quad nT \leq \zeta < nT + T$$

$$\underline{w}(\zeta) = \underline{w}(nT) \quad\quad nT \leq \zeta < nT + T$$

Let $\underline{F}_1 = \int_{nT}^{nT+T} e^{\underline{F}(nT+T-\zeta)} d\zeta$ with shortened notation, equation (V.6)

becomes $\underline{x}(n+1) = (\underline{I} + \underline{F}T)\, \underline{x}(n) + \underline{F}_1 (\underline{D}_v\, \underline{v}(n) + \underline{D}_w\, \underline{w}(n) + \underline{D}_o)$ (V.7)

or, $\underline{x}(n+1) = (\underline{I} + \underline{H}T + \sum_{i=1}^{N_u} u_i(n)\, \underline{E}_{u_i} T + \sum_{i=1}^{N_v} v_i(n)\, \underline{E}_{v_i} T)\, \underline{x}(n) +$

$$\underline{F}_1\, \underline{D}_v\, \underline{v}(n) + \underline{F}_1\, \underline{D}_w\, \underline{w}(n) + \underline{F}_1\, \underline{D}_o \tag{V.8}$$

If we let

$$\underline{A} = \underline{I} + \underline{H}T$$

$$\underline{B}_{u_i} = \underline{E}_{u_i} T$$

$$\underline{B}_{v_i} = \underline{E}_{v_i} T \tag{V.9}$$

$$\underline{C}_v = \underline{F}_1\, \underline{D}_v$$

$$\underline{C}_w = \underline{F}_1\, \underline{D}_w$$

$$\underline{C}_o = \underline{F}_1\, \underline{D}_o$$

and change n by k, we obtain

$$\underline{x}(k+1) = (\underline{A} + \sum_{i=1}^{N_u} u_i(k)\, \underline{B}_{u_i} + \sum_{i=1}^{N_v} v_i(k)\, \underline{B}_{v_i})\, \underline{x}(k) +$$

$$\underline{C}_w\, \underline{w}(k) + \underline{C}_v\, \underline{v}(k) + \underline{C}_o \tag{V.10}$$

The above is a general discrete-time belinear system equation. Thus, it is possible to approximate to the first order a continuous bilinear plant by an equivalent discrete-time bilinear one.

The general discrete-time plant equation is rewritten as follows [1]:

$$\underline{x}(k+1) = [\underline{A} + \sum_{i=1}^{N_u} u_i(k)\, \underline{B}_{u_i} + \sum_{i=1}^{N_v} v_i(k)\, \underline{B}_{v_i}]\, \underline{x}(k) + \underline{C}_v\, \underline{v}(k) +$$

$$\underline{C}_w\, \underline{w}(k) + \underline{C}_o \tag{V.11}$$

where,

$\underline{x}(k)$ = state vector at time (k)

$\underline{x}(k+1)$ = state vector at time (k+1)

$u_i(k)$ = multiplicative control input (i) at time (k)
($i = 1, \ldots, N_u$)

$v_i(k)$ = multiplicative and additive control input (i) at time (k)
($i = 1, \ldots, N_v$)

$\underline{w}(k)$ = additive control input vector at time (k)

$\underline{C}_o$ = vector of constant inputs to the system

$\underline{A}$, $\underline{B}_{u_i}$, $\underline{B}_{v_i}$, $\underline{C}_v$, $\underline{C}_w$, $\underline{C}_o$ = constant matrices of respective suitable dimensions

If we want the plant to operate at state vector $\underline{X}_{(d)}$ and suppose that this state can be made an equilibrium state of the system, i.e.

$$\underline{x}_d = [\underline{I} - \underline{A} - \sum_{i=1}^{N_u} u_{id}\,\underline{B}_{u_i} - \sum_{i=1}^{N_v} v_{id}\,\underline{B}_{v_i}]^{-1}\{[\underline{C}_v\ \underline{C}_w]\times \begin{bmatrix} \underline{v}_d \\ \underline{w}_d \end{bmatrix} + \underline{C}_o\} \tag{V.12}$$

The above expression is obtained by solving equation (V.11) with the left hand side set to zero and $\underline{x}(k)$, $\underline{u}(k)$, $\underline{w}(k)$, $\underline{v}(k)$ respectively replaced by $\underline{x}_d$, $\underline{u}_d$, $\underline{w}_d$, $\underline{v}_d$. $\underline{u}_d$, $\underline{w}_d$, $\underline{v}_d$ are control inputs to keep the system at equilibrium.

Define the error state variables as follows:

$$\delta\underline{X}(k) = \underline{X}(k) - \underline{X}_d$$

$$\text{or}\quad \underline{X}(K) = \underline{X}_d + \delta\underline{X}(k) \tag{V.13}$$

Substituting (V.13) into equation (V.11), we obtain:

$$\begin{aligned} \delta\underline{X}(k+1) = {} & [\underline{A} + \sum_{i=1}^{N_u} u_i(k)\,\underline{B}_{u_i} + \sum_{i=1}^{N_v} v_i(k)\,\underline{B}_{v_i}]\,\delta\underline{X}(k) + \\ & [\sum_{i=1}^{N_u} (u_i(k) - u_{id})\,\underline{B}_{u_i} + \sum_{i=1}^{N_v} (v_i(k) - v_{id})\,\underline{B}_{v_i}]\,\underline{X}_d + \\ & [\underline{C}_v\ \underline{C}_w]\begin{bmatrix} \underline{v} - \underline{v}_d \\ \underline{w} - \underline{w}_d \end{bmatrix} \end{aligned} \tag{V.14}$$

Let $\underline{A}_d = \underline{A} + \sum_{i=1}^{N_u} u_{id}\,\underline{B}_{u_i} + \sum_{i=1}^{N_v} v_{id}\,\underline{B}_{v_i}$

$$\delta u_i(k) = u_i(k) - u_{id}$$
$$\delta v_i(k) = v_i(k) - v_{id}$$
$$\delta w_i(k) = w_i(k) - w_{id}$$

Equation (V.14) can be rewritten as:

$$\underline{\delta X}(k+1) = [\underline{A}_d + \sum_{i=1}^{N_u} \delta u_i(k)\, \underline{B}_{u_i} + \sum_{i=1}^{N_v} \delta v_i(k)\, \underline{B}_{v_i}]\, \delta\underline{X}(k) + \underline{D} \begin{bmatrix} \delta\underline{u}(k) \\ \delta\underline{v}(k) \\ \delta\underline{w}(k) \end{bmatrix} \qquad (V.15)$$

where,

$$\underline{D} = [D_u \mid D_v \mid D_w]$$

$$\underline{D}_u = [\underline{B}_{u_1} \cdot \underline{x}_d \mid \underline{B}_{u_2} \cdot \underline{x}_d \mid . \; . \; . \; . \mid \underline{B}_{u_{N_u}} \cdot \underline{x}_d]$$

$$\underline{D}_v = [\underline{B}_{v_1} \cdot \underline{x}_d + \underline{C}_{v_1} \mid . \; . \; . \; . \mid \underline{B}_{v_{N_v}} \cdot \underline{x}_d + \underline{C}_{v_{N_v}}]$$

$$\underline{D}_w = \underline{C}_w$$

Hence, a chosen $\underline{x}_d \neq 0$ generally introduced additive control-variation terms for all the multiplicative control variables, and there is no "separation" of additive/multiplicative control inputs (except that $\underline{w}$, if present, remains only additive).

If $\underline{\delta X}(o)$ is the initial error state, and if we can find a control sequence of length N such that $\delta\underline{X}(N) = \underline{o}$, then defining

$$\underline{M}_j = [A_d + \sum_{i=1}^{N_u} \delta u_i(j)\, \underline{B}_{u_i} + \sum_{i=1}^{N_v} \delta v_i(j)\, \underline{B}_{v_i}] \qquad (V.16)$$

From (V.15), we can write

$$\underline{\delta X}(1) = \underline{M}_o \delta\underline{X}(o) + \underline{D} \begin{bmatrix} \underline{\delta u}(o) \\ \underline{\delta v}(o) \\ \underline{\delta w}(o) \end{bmatrix}$$

$$\underline{\delta X}(2) = \underline{M}_1 \delta\underline{X}(1) + \underline{D} \begin{bmatrix} \underline{\delta u}(1) \\ \underline{\delta v}(1) \\ \underline{\delta w}(1) \end{bmatrix} \qquad \text{(V.17a)}$$

$$\vdots$$

$$\underline{\delta X}(N) = 0 = \underline{M}_{N-1} \delta\underline{X}(N-1) + \underline{D} \begin{bmatrix} \underline{\delta u}(N-1) \\ \underline{\delta v}(N-1) \\ \underline{\delta w}(N-1) \end{bmatrix}$$

or, eliminating all $\underline{\delta X}(j)$ for $j > o$, we obtain:

$$-[\underline{M}_{N-1}\,\underline{M}_{N-2} \cdots \underline{M}_o]\,\underline{\delta X}(o) = [\underline{D} \mid \underline{M}_{N-1}\,\underline{D} \mid \cdots \mid (\underline{M}_{N-1} \cdots \underline{M}_1)\,\underline{D}] \times$$

$$\begin{bmatrix} [\underline{\delta u}^T(N-1)\ \underline{\delta v}^T(N-1)\ \underline{\delta w}^T(N-1)]^T \\ \vdots \\ [\underline{\delta u}^T(o)\ \underline{\delta v}^T(o)\ \underline{\delta w}^T(o)]^T \end{bmatrix} \qquad \text{(V.17b)}$$

Thus the problem is to

a) find a sequence of controls which satisfy the above equation, and

b) optimize, in some sinese, the performance of the system.

Condition (a) is the controllability problem, and we shall show that this has solution, in general; even if only one control variable is available and it is both additive and multiplicative. The more difficult question is: how to use the freedom of the richer control structure to optimize performance (b)?

Suppose that we would like to minimize the vector length of the solution control sequence appearing on the right hand side of the equation (V.17), i.e.,

$$\min \left[\sum_{i=1}^{N-1} \left(\alpha_u ||\underline{\delta u}(i)||^2 + \alpha_v ||\underline{\delta v}(i)||^2 + \alpha_w ||\underline{\delta w}(i)||^2\right)\right]^{1/2}$$

Assuming that, minimization of the above expression is directly related to the elapsed time for the control efforts to bring the plant back to its equilibrium state, in other words, the faster the system reaches its equilibrium, the less power it consumes (in a physical system, $\underline{\delta u}$, $\underline{\delta v}$, $\underline{\delta w}$ represents power consumed).

Pick one of the control variables, say δv_1, and initially set it to zero

$$\delta v_1(j) = 0 \qquad j = 0, 1, \ldots, N-1$$

Then use the remaining control variables to establish a set of state-transition matrices $\tilde{M}_o, \ldots, \tilde{M}_{N-1}$ such that each $\tilde{M}_j$ has a "fast" eigenvector $\underline{m}_j$ (i.e., $\underline{\tilde{M}}_j\, m_j = z_j\, \underline{m}_j$ with $|z_j| << 1$) and the set $\{\underline{m}_o, m_1, \ldots, \underline{m}_{N-1}\}$ are as nearly as orthogonal as possible (assuming that $N = n =$ dimension of the state space) i.e., we want them to be a basis in the state space

where $$\underline{\tilde{M}}_j = [A_d + \sum_{i=1}^{N_u} \delta u_i(j)\, \underline{B}_{u_i} + \sum_{i=2}^{N_v} \delta v_i(j)\, \underline{B}_{v_i}] \qquad \text{(V.18)}$$

The idea behind the above algorithm is that, to minimize the control-variation efforts, we would like to have δv as small as possible, in some sense. This is achieved if such a set of "fast" eigenvectors can be synthesized using the other control variables (in particular, the

the purely multiplicative ones, the u's, may not involve much energy comsumption so their magnitudes need not be minimized. This is true for a solar-assisted heat pump system). Thus, if δv_1 can be kept small, the perturbations caused by $\delta v_1 \neq 0$ on the eigenvectors $\underline{m}_j$ can be small so the synthesis will still be fast to a close approximation.

To implement the algorithim as closed-loop control switching, the current measured state vector would be analysized in the basis of the fast eigenvectors, e.g.:

$$\delta \underline{X}_o = \gamma_o \underline{m}_o + \gamma_1 \underline{m}_1 + \ldots + \gamma_{n-1} \underline{m}_{n-1} \qquad \text{(V.19)}$$

Identify the largest component, let us say γ_i,

$$|\gamma_i| \geq |\gamma_j| \quad \text{all } j \neq i$$

Adjust the "other" control variables to produce state transition matrix $\underline{\tilde{M}}_i$.

Then, after one sample period, if no further state disturbance occurs, the next state would be:

$$\underline{\delta X}_1 = \underline{\tilde{M}}_i \, \underline{\delta X}_o + \underline{D} \begin{vmatrix} \underline{\delta u}(o) \\ \underline{\delta v}(o) \\ \underline{\delta w}(o) \end{vmatrix}$$

$$\underline{\delta X}_1 = \sum_{\substack{j=o \\ j\neq i}}^{n-1} \gamma_j \underline{\tilde{M}}_i m_j + \gamma_i z_i \underline{m}_i + \underline{D}_u \, \delta \underline{u} + \underline{D}'_v \, \underline{\delta v}' + \underline{D}_w \, \underline{\delta w} +$$

$$(B_{v_1} \underline{X}_d + C_{v_1}) \delta_{v_1} \qquad \text{(V.20)}$$

where $\underline{D}'_v = \{\text{last } (N_v - 1) \text{ columns of } \underline{D}_v\}$

$$\underline{\delta v}' = \{\text{last } (N_v-1) \text{ components of } \underline{\delta v}\}$$

Since $|z_i|<<1$, the dominant $\underline{m}_i$ component of $\underline{\delta X}_o$ has been greatly reduced by the action along the "fast" eigenvector direction.

The additive terms in $\underline{\delta u}$, $\underline{\delta w}$, and $\delta v_j (j \neq 1)$ may be viewed as constant forcing inputs using the sample period, causing some further incremental displacement of the state. The remaining additive control term δv_1 should be set at the appropriate value for "deadbeat control". For example, suppose we ignore the other additive terms for the present, and set

$$\underline{\delta X}_1 = \underline{\bar{M}}_i \; \underline{\delta X}_o + \underline{\Gamma} \delta v_1$$

Then $\underline{\delta X}_o \rightarrow \underline{o}$ in one sample period if the equation $\underline{o} = \underline{\tilde{M}}_i \delta X_o + \underline{\Gamma} \delta v_1$ has a solution $\delta v_1(o)$. This is possible if and only if $\underline{\delta X}_o = \underline{\tilde{M}}_i^{-1} \underline{\Gamma}$ for some scales α i.e., $\underline{\delta X}_o$ happens to lie along the vector $\underline{\tilde{M}}_i^{-1} \underline{\Gamma}$ in state space. If this is the case, then choose δv_1 equal to $-\alpha$.

Likewise, $\underline{\delta X}_o \rightarrow \underline{o}$ in two sample periods with an open-loop control sequence $\{\delta v_1(o), \delta v_1(1)\}$ if and only if the following equation has a solution sequence

$$\underline{o} = \underline{\tilde{M}}_{i(1)} \; \underline{\delta X}(1) + \underline{\Gamma} \delta v_1(1)$$

or $$\underline{o} = \underline{\tilde{M}}_{i(1)} [\underline{\tilde{M}}_{i(o)} \; \underline{\delta X}(o) + \underline{\Gamma} \; \delta v_1(o)] + \underline{\Gamma} \; \delta v_1(1)$$

The above equation has solution if and only if

$$\underline{\delta X}(o) = \alpha \underline{\tilde{M}}_{i(o)}^{-1} \underline{\Gamma} + \beta \underline{\tilde{M}}_{i(o)}^{-1} \; \underline{\tilde{M}}_{i(1)}^{-1} \underline{\Gamma}$$

for some α, β the two-step open loop sequence would be $\delta v_1(o) = -\alpha$ $\delta v_1(1) = -\beta$.

In general, for an n-step dead-beat control sequence, $\underline{\delta X}(o)$ has to be in the following direction:

$$\underline{\delta X}(o) = \alpha \underline{\tilde{M}}^{-1}_{i(o)} \underline{\Gamma} + \sum_{l=1}^{n-1} \left[\prod_{k=0}^{l} \tilde{M}^{-1}_{i(k)} \right] \beta_l \underline{\Gamma} \tag{V.21}$$

where $\delta v_1(o) = -\alpha$

and $\delta v_1(l) = \beta_l \qquad l = 1, \ldots, n-1$

Note, however, that only $\delta v_1(o)$ would be used in closed-loop control (eqn. (V.20)), after one sample period a new feed-back state-measurement would be available. Nonetheless, all β's have to be calculated in order to find α.

The other additive forcing terms do not really complicate the matter very much. If we have, for example,

$$\underline{o} = \underline{\tilde{M}}_{i(o)} \underline{\delta X}(o) + \underline{\Gamma} \delta v_1(o) + \underline{\Gamma}_w \delta w(o) \tag{V.22}$$

we must have

$$\underline{\delta X}(o) = \alpha \tilde{M}^{-1}_{i(o)} \underline{\Gamma} - \underline{\tilde{M}}^{-1}_{i(o)} \underline{\Gamma}_w \delta w(o)$$

then $\delta v_1(o) = -\alpha$ is chosen, here we have the freedom of adjusting $\delta w(o)$ for a proper α. However, if $\delta u(o)$ takes the place of $\delta w(o)$ in (V.22), α cannot be varied because $\delta u(o)$ has been fixed by the process of establishing $\underline{\tilde{M}}_{i(o)}$.

From a slightly different point of view, equation (V.17) defines an open-loop sequence of control such that:

$$-[\underline{M}_{i(n-1)} \underline{M}_{i(n-2)} \cdots \underline{M}_{i(o)}] \underline{\delta X}(o) = [\underline{D} \mid \underline{M}_{i(n-1)} \underline{D} \mid \ldots$$

$$|\underline{M}_{i(n-1)} \cdots \underline{M}_{i(1)}\ \underline{D}] \times \begin{bmatrix} [\underline{\delta u}^T(n-1)\ \ \underline{\delta v}^T(n-1)\ \ \underline{\delta w}^T(n-1)]^T \\ \vdots \\ [\underline{\delta u}^T(o)\ \ \underline{\delta v}^T(o)\ \ \underline{\delta w}^T(o)]^T \end{bmatrix} \qquad (V.23a)$$

Here $\underline{M}_{N-1}$, $\underline{M}_{N-2}$, etc. ... in eqn. (V.17) are replaced by $\underline{M}_{i(n-1)}$, $\underline{M}_{i(n-2)}$, etc. ...

Assuming that $\delta v_1(j)$ ($j = 1, \ldots, n$) is negligible.

Equation (V.23a) can be rewritten as (compare(V.16) and (V.18)):

$$-[\tilde{\underline{M}}_{i(n-1)}\ \tilde{\underline{M}}_{i(n-2)} \cdots \tilde{\underline{M}}_{i}(o)]\underline{\delta X}(o) = [\underline{D}\ |\ \tilde{\underline{M}}_{i(n-1)}\underline{D}|\ \cdots$$

$$|\tilde{\underline{M}}_{i(n-1)} \cdots \tilde{\underline{M}}_{i(1)}\ \underline{D}] \times \begin{bmatrix} [\underline{\delta u}^T(n-1)\ \ \underline{\delta v}^T(n-1)\ \ \underline{\delta w}^T(n-1)]^T \\ \vdots \\ [\underline{\delta u}^T(o)\ \ \underline{\delta v}^T(o)\ \ \underline{\delta w}^T(o)]^T \end{bmatrix} \qquad (V.23b)$$

$i(o)$, $i(1)$, etc. ... denote the index of the particular "fast eigenvector matrix" that would be selected at each sampling period. That is, in the first sample, solving equation (V.23) give us $\delta v_1(o)$ (we obtain $\delta v_1(j)$, $j = 0, \ldots, n-1$, but only $\delta v_1(o)$ is needed in this first step) with all other controls having been given in the process of establishing $\tilde{\underline{M}}_{o(n-1)}$, $\tilde{\underline{M}}_{o(n-2)}$, ... $\tilde{\underline{M}}_{o(o)}$. Using equation (V.17a), the state vector at the second step is calculated:

$$\underline{\delta X}(1) = \underline{M}_o\ \underline{\delta X}(o) + \underline{D} \begin{bmatrix} \underline{\delta u}(o) \\ \underline{\delta v}(o) \\ \underline{\delta w}(o) \end{bmatrix} \qquad (V.17a)$$

where $\underline{M}_o = \tilde{\underline{M}}_{o(o)} + B_{v_1}\delta v_1(o)$

The control sequence length now is (n-1), solve for $\delta v_1(1)$ by using the modified equation (V.23b)

$$-[\underline{\tilde{M}}_{1(n-1)}\,\underline{\tilde{M}}_{1(n-2)}\cdots\underline{\tilde{M}}_{1(1)}]\underline{\delta X}(1) = [\underline{D} \mid \underline{\tilde{M}}_{1(n-1)}\,\underline{D} \mid \cdots$$

$$|\tilde{M}_{1(n-1)}\cdots\underline{\tilde{M}}_{1(2)}\,\underline{D}] \times \left|\begin{matrix} [\underline{\delta u}^T(n-1) & \underline{\delta v}^T(n-1) & \underline{\delta w}^T(n-1)]^T \\ & \vdots & \\ [\underline{\delta u}^T(1) & \underline{\delta v}^T(1) & \underline{\delta w}^T(1)]^T \end{matrix}\right|$$

The state vector at the third sample is

$$\underline{\delta X}(2) = \underline{M}_1\,\underline{\delta X}(1) + \underline{D}\left|\begin{matrix}\underline{\delta w}(1)\\ \underline{\delta v}(1)\\ \underline{\delta w}(1)\end{matrix}\right|$$

where $\underline{M}_1 = \underline{\tilde{M}}_{1(1)} + \underline{B}_{v_1}\delta v_{1(1)}$

And then repeat the process by using equation (V.23b) to calculate the control $\delta v_1(2)$, etc. ... From the equation (V.23b), we can write:

$$-\underline{\delta X}(o) = \bar{\underline{\underline{\Phi}}}\left|\begin{matrix}\delta v_1(n-1)\\ \delta v_1(n-2)\\ \vdots\\ \delta v_1(o)\end{matrix}\right| + \text{(a vector independent of } \delta v_1) \qquad \text{(V.24)}$$

where $\bar{\underline{\underline{\Phi}}} = [\underline{\tilde{M}}^{-1}_{i(o)}\,\underline{\tilde{M}}^{-1}_{i(1)}\cdots\underline{\tilde{M}}^{-1}_{i(n-1)}\,\underline{D}_{v_1} | \cdots | \underline{\tilde{M}}^{-1}_{i(o)}\,\underline{D}_{v_1}]$

$\underline{D}_{v_1}$ = the part of $\underline{D}_v$ that multiplies δv_1 (it was called $\underline{\Gamma}$ in equation V.21).

$\underline{D}_{v_1} = \underline{B}_{v_1}\,\underline{X}_d + \underline{C}_{v_1}$

The question is what properties the matrix $\bar{\underline{\underline{\Phi}}}$ should have in order to minimize the solution $[\delta v_1(n-1)\ \delta v_1(n-2)\ \dots\ \delta v_1(o)]^T$? Since we are inverting this matrix to solve for the δv_1's, the matrix should be nonsingular and well-conditioned. That is, the columns of $\bar{\underline{\underline{\Phi}}}$ must be linearly independent, should be as nearly orthogonal as possible, and $\bar{\underline{\underline{\Phi}}}$ should be uniformly large in some sense in order for the δv_1's to be minimized. In the sense of the spectral norm, the eigenvalues of $\bar{\underline{\Phi}}$ should all be large in magnitude.

In general, these conditions may perhaps all be met, as will be proven in the following, if and only if $\underline{\tilde{M}}_{i(o)}$, $\underline{\tilde{M}}_{i(1)}$, etc. each has one "fast" eigenvector and these eigenvectors are nearly orthogonal.

The sufficient condition of the above statement will be proven as follows, for the necessary condition it is similar.

Let the last column of $\bar{\underline{\underline{\Phi}}}$ be $\underline{\phi}_n$, it can be written as:

$$\underline{\phi}_n = \underline{\tilde{M}}^{-1}_{i(o)}\ \underline{D}_{v_1} \quad \text{or} \quad \underline{D}_{v_1} = \underline{\tilde{M}}_{i(o)}\ \underline{\phi}_n$$

$\underline{\phi}_n$ can be resolved in the basis of the fast eigenvectors $\{\underline{m}_{i(o)}, \underline{m}_{i(1)}, \dots, \underline{m}_{i(n-1)}\}$

$$\underline{\phi}_n = \alpha_{n,0}\ \underline{m}_{i(o)} + \alpha_{n,1}\ \underline{m}_{i(1)} + \dots + \alpha_{n,n-1}\ \underline{m}_{i(n-1)}$$

or

$$\underline{\tilde{M}}_{i(o)}\ \underline{\phi}_n = \alpha_{n,0}\ \underline{\tilde{M}}_{i(o)}\ \underline{m}_{i(o)} + \alpha_{n,1}\ \underline{\tilde{M}}_{i(o)}\ \underline{m}_{i(1)} + \dots$$

$$\underline{D}_{v_1} = \alpha_{n,0}\ z_{i(o)}\ \underline{m}_{i(o)} + \alpha_{n,1}\ \underline{\tilde{M}}_{i(o)}\ \underline{m}_{i(1)} + \dots$$

Because $\underline{D}_{v_1}$ is a constant vector, and $z_{i(o)}$ is the fast eigenvalue, hence:

$$\alpha n,0\ z_{i(o)} = \text{constant}$$

$z_{i(o)}$ is very small, thus $\alpha_{n,o}$ must be large. Therefore, the component of $\underline{\phi}_n$ along the direction $\underline{m}_{i(o)}$ must be large.

Likewise, the next-to-last column of $\bar{\underline{\Phi}}$ is:

$$\underline{\phi}_{n-1} = \tilde{\underline{M}}^{-1}_{i(o)} \tilde{\underline{M}}^{-1}_{i(1)} \underline{D}_{v_1}$$

or,

$$\tilde{\underline{M}}_{i(1)} \tilde{\underline{M}}_{i(o)} \underline{\phi}_{n-1} = \underline{D}_{v_1}$$

Similarly to the previous case, the component of vector $\tilde{\underline{M}}_{i(o)} \underline{\phi}_{n-1}$ along the direction $\underline{m}_{i(1)}$ must be large, or $\underline{\phi}_{n-1}$ must have a large component in the direction of vector $\tilde{\underline{M}}^{-1}_{i(o)} \underline{m}_{i(1)}$ which is not close to the direction of $\underline{m}_{i(o)}$. Suppose it were, we can write:

$$\tilde{\underline{M}}^{-1}_{i(o)} \underline{m}_{i(1)} = \alpha \underline{m}_{i(o)}$$

or

$$\underline{m}_{i(1)} = \alpha \tilde{\underline{M}}_{i(o)} \underline{m}_{(o)} = \alpha z_{i(o)} \underline{m}_{i(o)}$$

So, vectors $\underline{m}_{i(1)}$ and $\underline{m}_{i(o)}$ are of close direction, this contradicts the assumption that all the eigenvectors $\underline{m}_{i(j)}$, $\underline{m}_{i(k)}$, $(j,\ k = 0,\ \ldots,\ n-1)$ are nearly orthogonal.

Therefore, direction $\tilde{\underline{M}}^{-1}_{i(o)} \underline{m}_{i(1)}$ is not close to direction $\underline{m}_{i(o)}$, matrix $\tilde{\underline{M}}^{-1}_{i(o)}$ can be thought of as amplifying the component of $\underline{m}_{i(1)}$ that is in the direction of $\underline{m}_{i(o)}$, but that component is very small because $\underline{m}_{i(1)}$ and $\underline{m}_{i(o)}$ are nearly-orthogonal. In other words, column $\underline{\phi}_n$ is nearly-orthogonal to column $\underline{\phi}_{n-1}$. Continuing, the second-to-last column of $\bar{\underline{\Phi}}$ is:

$$\underline{\phi}_{n-2} = \tilde{\underline{M}}^{-1}_{i(o)} \tilde{\underline{M}}^{-1}_{i(1)} \tilde{\underline{M}}^{-1}_{i(2)} \underline{D}_{v_1}$$

or $\underline{\tilde{M}}_{i(2)} \, \underline{\tilde{M}}_{i(1)} \, \underline{\tilde{M}}_{i(o)} \, \underline{\phi}_{n-2} = \underline{D}_{v_1}$

Hence, $\underline{\tilde{M}}_{i(1)} \, \underline{\tilde{M}}_{i(o)} \, \underline{\tilde{\phi}}_{n-2}$ has a large component along the direction of $\underline{m}_{i(2)}$ or $\underline{\tilde{\phi}}_{n-2}$ has a large component along the direction of $\underline{\tilde{M}}^{-1}_{i(o)} \, \underline{\tilde{M}}^{-1}_{i(1)} \, \underline{m}_{i(2)}$. $\underline{\tilde{M}}^{-1}_{i(o)}$ and $\underline{\tilde{M}}^{-1}_{i(1)}$ respectively amplify the components of $\underline{m}_{i(2)}$ that are in the direction of $\underline{m}_{i(o)}$ and $\underline{m}_{i(1)}$ respectively, but these components are small because $\underline{m}_{i(2)}$ is nearly orthogonal to $\underline{m}_{i(o)}$ and $\underline{m}_{i(1)}$, therefore, direction $\underline{M}^{-1}_{i(o)} \, \underline{M}^{-1}_{i(1)} \, \underline{m}_{i(2)}$ is not close to the direction of $\underline{m}_{i(o)}$ and $\underline{m}_{i(1)}$, or the direction of the second-to-last column of $\bar{\underline{\underline{\Phi}}}$, $\underline{\phi}_{n-2}$, is not close to the directions of $\underline{\phi}_{n-1}$ and $\underline{\phi}_{n}$, and is nearly-orthogonal to $\underline{\phi}_{n-1}$ and $\underline{\phi}_{n}$ (because components of $\underline{\phi}_{n}$ along the direction of $\underline{\phi}_{n}$ and $\underline{\phi}_{n-1}$ are very small).

Similarly, we can prove that all the directions of all the columns of $\bar{\underline{\underline{\Phi}}}$ are not close to one another, or they are linearly independent, and are relatively orthogonal to one another.

Let U be a set of basis rectors which are in the direction of $\underline{m}_{i(o)}$, $\underline{\tilde{M}}^{-1}_{i(o)} \, \underline{m}_{i(1)}$, $\underline{\tilde{M}}^{-1}_{i(o)} \, \underline{\tilde{M}}_{i(1)} \, \underline{m}_{i(2)}$, etc. ..., respectively. Because these vectors are nearly orthogonal, if we express columns of $\bar{\underline{\underline{\Phi}}}$ in this basis, $\bar{\underline{\underline{\Phi}}}$ will be composed of mainly diagonal elements. These diagonal elements of $\bar{\underline{\underline{\Phi}}}$ will be its component in the direction of the basis vectors U and these elements are large, hence, all of the eigenvalues of $\bar{\underline{\underline{\Phi}}}$ are large or $\bar{\underline{\underline{\Phi}}}$ is uniformly large in terms of spectral norm.

Solving equation (IV.24), we obtain the entire deadbeat open-loop sequence. But the practical problem of finding the control sequence δv_1 remains difficult, since, as we see previously, the matrix $\bar{\underline{\underline{\Phi}}}$ depends on the matrices $\tilde{M}_{i(j)}$ ($i = 1, \ldots, n$; $j = 1, \ldots, n$), which, in turn, changes at every sampling step. So this would mean inverting a different $\bar{\underline{\underline{\phi}}}$ at each sample and this is computationally infeasible with microprocessor.

Until now we have assumed that null-controllability of the system is possible with the two-step process of:

a) First set $\delta v_1(j) = 0 \quad j = 0, 1, \ldots, n$; then find n different sets of values for all the other δu's, δv's that give n relatively orthogonal fast eigenvectors for the corresponding state-transition matrices.

b) Then find δv_1 values that tend toward deadbeat control closed loop, assuming that these values are small and do not significantly affect the fast eigenvectors found in (a).

In the following, we will examine this assumption to see whether it is indeed possible to have null-controllability when δv_1 in fact appears both multiplicatively and additively.

First consider the case n = 2, assuming that $\underline{\delta X}(2) = \underline{0}$ can be achieved. From eqn. (V.17a), we can write:

$$\underline{\delta X}(1) = \underline{M}_{i(o)}\underline{\delta X}(o) + \underline{D}\begin{bmatrix}\underline{\delta u}(o)\\ \underline{\delta v}(o)\\ \underline{\delta w}(o)\end{bmatrix}$$

and

$$\underline{\delta X}(2) = \underline{0} = \underline{M}_{i(1)}\underline{\delta X}(1) + \underline{D}\begin{bmatrix}\underline{\delta u}(1)\\ \underline{\delta v}(1)\\ \underline{\delta w}(1)\end{bmatrix}$$

or

$$\underline{0} = \underline{M}_{i(1)}[\underline{M}_{i(o)}\underline{\delta X}(o) + \underline{D}\begin{bmatrix}\underline{\delta w}(o)\\ \underline{\delta v}(o)\\ \underline{\delta w}(o)\end{bmatrix}] + \underline{D}\begin{bmatrix}\underline{\delta u}(1)\\ \underline{\delta v}(1)\\ \underline{\delta w}(1)\end{bmatrix}$$

or

$$\underline{0} = \underline{M}_{i(1)}\underline{M}_{i(o)}\underline{\delta X}(o) + \underline{M}_{i(1)}\underline{D}_{v_1}\ \delta v_1(o) + \underline{D}_{v_1}\delta v_1(1) +$$

$$\text{(a vector independent of } \delta v_1(o)) \qquad (V.25)$$

compare equations (V.16) and (V.18), we obtain

$$\underline{M}_{i(k)} = \tilde{\underline{M}}_{i(k)} + \underline{B}_{v_1}\delta v_1(k) \qquad (V.26)$$

Substituting (V.26) into (V.25)

$$\underline{0} = [\tilde{\underline{M}}_{i(1)} + \underline{B}_{v_1}\delta v_1(1)]\ [\tilde{\underline{M}}_{i(o)} + \underline{B}_{v_1}\delta v_1(o)]\underline{\delta X}(o) +$$

$$[\tilde{\underline{M}}_{i(1)} + \underline{B}_{v_1}\delta v_1(1)]\ \underline{D}_{v_1}\delta v_1(o) + \underline{D}_{v_1}\delta v_1(1) +$$

$$\text{(a vector independent of } \delta v_1(o)).$$

or

$$\underline{0} = -\underline{\delta X}(o) - [\tilde{\underline{M}}_{i(o)} + \underline{B}_{v_1}\ \delta v_1(o)]^{-1}\ \underline{D}_{v_1}\delta v_1(o)$$

$$-[\tilde{\underline{M}}_{i(o)} + \underline{B}_{v_1}\delta v_1(o)]^{-1}\ [\tilde{\underline{M}}_{i(1)} + \underline{B}_{v_1}\delta v_1(1)]^{-1}\underline{D}_{v_1}\delta v_1(1) +$$

$$\text{(a vector dependent of } \delta u\text{'s, } \delta v\text{'s, } \delta w\text{'s and other than}$$

$$\delta v_1(o),\ \delta v_1(1)) \qquad (V.27)$$

(V.27) is obtained by assuming that $\delta v_1(o)$ and $\delta v_1(1)$ are very small and can be neglected in the last term of the equation.

Equation (V.27) is true if and only if we can find $\delta v_{1(o)}$ and $\delta v_1(1)$ such that the vector function

$$\underline{C} = [\tilde{\underline{M}}_{i(o)} + \underline{B}_{v_1}\delta v_1(o)]^{-1}\underline{D}_{v_1}\delta v_1(o) + [\tilde{\underline{M}}_{i(o)} + \underline{B}_{v_1}\delta v_1(o)]^{-1}\ x$$

$$[\underline{\tilde{M}}_{i(1)} + \underline{B}_{v_1}\delta v_1(1)]^{-1}\underline{D}_{v_1}\delta v_1(1) \qquad (V.28)$$

covers the entire plane, i.e., values of $\delta v_1(o)$ and $\delta v_1(1)$ can always be found such that this function equals the arbitrary vector given by the other terms in equation (V.27). $\underline{C}$ can be written as the sum of the two vector functions:

$$\underline{C} = \underline{s}[\delta v_1(o)] + \underline{t}[\delta v_1(o), \delta v_1(1)] \qquad (V.29)$$

The locus of $\underline{s}$ will be examined first. $\underline{s}$ is a curve in R^2, passing through the origin when $\delta v_1(o) = 0$ and asymptotic to the straight line $\underline{\tilde{M}}_{i(o)}^{-1}\,\underline{D}_{v_1}\,\delta v_1(o)$ as $\delta v_1(o) \to 0$. For $\delta v_1(o) \neq 0$, there may be up to two finite, real values of $\delta v_1(o)$ for which $\underline{\tilde{M}}_{i(o)} + \underline{B}_{v_1}\cdot\delta v_1(o)$ is singular. Let $v_o{}^*$ be such a value. Then as $\delta v_1(o) \to v_o^*$, $\underline{s}[\delta v_1(o)] \to \infty$ asymptotic to the null space of $[\underline{\tilde{M}}_{i(o)} + \underline{B}_{v_1}\ v_1{}^*]$, which is denoted as $N_o{}^*$ (Fig. V.1).

If there are two singular values $v_o{}^*$ and $v_1{}^{**}$ with, say $0 < v_o{}^* < v_o{}^{**}$ then the locus consists of two curves, each tending to ∞ as $\delta v_1(o) \to v_o{}^*$ and $v_o{}^{**}$, respectively (Fig. V.2).

Assuming B_{v_1} is nonsingular, as $\delta v_1(o) \to \pm\infty$, $\underline{s} \to \underline{B}_{v_1}^{-1}\,\underline{D}_{v_1}$, if $\underline{B}_{v_1}$ is singular:

$$\underline{s} = [\underline{\tilde{M}}_{i(o)} + \underline{B}_{v_1}\delta v_1(o)]^{-1}\,\underline{D}_{v_1}\delta v_1(o) \qquad (V.30)$$

or $\quad [\underline{\tilde{M}}_{i(o)} + \underline{B}_{v_1}\delta v_1(o)]\underline{s} = \underline{D}_{v_1}\delta v_1(o)$

when $\delta v_1(o) \to \infty$, $\underline{s} \to \infty$, asymptotic to the null space of $\underline{B}_{v_1}$. Does the curve $\underline{s}$ loop and cut itself? I.e., can we have some $\underline{s}$ for which

$$\underline{s} = [\underline{\tilde{M}}_{i(o)} + \underline{B}_{v_1}\alpha]^{-1}\underline{D}_{v_1}(\alpha) = [\underline{\tilde{M}}_{i(o)} + B_{v_1}\beta]^{-1}\underline{D}_{v_1}\beta \qquad (V.31)$$

for $\alpha \neq \beta$, $\alpha < v_o{*}$, $\beta > v_o{*}$, α and β are two distinct values of $\delta v_1(o)$. (v.31) can be rewritten as:

$$[\underline{\tilde{M}}_{i(o)} + \underline{B}_{v_1}\alpha]\underline{s} - \underline{D}_{v_1}\alpha = 0 = [\tilde{M}_{i(o)} + \underline{B}_{v_1}\beta]\underline{s} - \underline{D}_{v_1}\beta$$

or $(\alpha - \beta)(\underline{B}_{v_1}\underline{s} - \underline{D}_{v_1}) = \underline{0}$

Since $\alpha \neq \beta$ $\quad \underline{B}_{v_1}\underline{s} - \underline{D}_{v_1} = \underline{0}$

This implies

$$\underline{\tilde{M}}_{i(o)}\underline{s} = 0 \qquad \text{(V.32)}$$

But this is impossible, since $\underline{\tilde{M}}_{i(o)}$ is a state-transition matrix for a discrete-time system and it cannot be singular. The only remaining possibility then is $\underline{s} = \underline{0}$ as a point of intersection, but from (V.31) this leads to $[\tilde{M}_{i(o)} + \underline{B}_{v_1}\alpha]^{-1}$ and $[\underline{\tilde{M}}_{i(o)} + \underline{B}_{v_1}\beta]^{-1}$ are singular. This is a contradiction since $[\underline{M}_{i(o)} + \underline{B}_{v_1}\alpha]$ and $[\underline{M}_{i(o)} + \underline{B}_{v_1}\beta]$ are both well-defined, finite matrices which are nonsingular and have nonsingular inverses $[\underline{\tilde{M}}_{i(o)} + \underline{B}_{v_1}\alpha]^{-1}$ and $[\underline{M}_{i(o)} + \underline{B}_{v_1}\beta]^{-1}$. Therefore, $\underline{s} = \underline{0}$ is not a point of intersection, either. In short, the curve does not loop and cut itself.

Now consider the function $\underline{t}$

$$\underline{t}[\delta v_1(o),\ \delta v_1(1)] = [\underline{\tilde{M}}_{i(o)} + \underline{B}_{v_1}\delta v_1(o)]^{-1}[\underline{\tilde{M}}_{i(1)} + \underline{B}_{v_1}\delta v_1(1)]^{-1} \cdot \underline{D}_{v_1}\,\delta v_1(1) \qquad \text{(V.33)}$$

for each fixed value of $\delta v_1(o)$, $\underline{t}$ is a function of $\delta v_1(1)$. $\underline{t}$ can be thought of as a vector displacement which is added to $\underline{s}[\delta v_1(o)]$ for each fixed value of $\delta v_1(o)$, $\underline{t}$ sweeps out a curve in the plane which passes through the point $\underline{s}[\delta v_1(o)]$ for $\delta v_1(1) = 0$. This curve goes to ∞ as $\delta v_1(1) \rightarrow v_1{}^*$ or $v_1{}^{**}$, where $\det[\tilde{\underline{M}}_{i(1)} + \underline{B}_{v_1} \delta v_1(1)] = 0$, the curve goes to infinity asymptotic to the null spaces $\underline{N}_1{}^*$, $\underline{N}_1{}^{**}$ of $[\tilde{\underline{M}}_{i(1)} + \underline{B}_{v_1} v_1{}^*] \times [\tilde{\underline{M}}_{i(o)} + \underline{B}_{v_1} \delta v_1(o)]$ or $[\tilde{\underline{M}}_{i(1)} + \underline{B}_{v_1} v_1{}^{**}]\, [\tilde{\underline{M}}_{i(o)} + \underline{B}_{v_1} \delta v_1(o)]$.

If $\underline{B}_{v_1}$ is singular, $\underline{t} \rightarrow \infty$ asymptotic to the line $[\underline{M}_{i(o)} + \underline{B}_{v_1} v_1(o)]^{-1} \times$ [null space of $\underline{B}_{v_1}$] when $\delta v_1(1) \rightarrow \pm\infty$.

If $\underline{B}_{v_1}$ is nonsingular, when $\delta v_1(1) \rightarrow \pm\infty$, $\underline{t} \rightarrow [\underline{M}_{i(o)} + \underline{B}_{v_1} v_1(o)]^{-1} \times \underline{B}_{v_1}^{-1} \underline{D}_{v_1}$.

From an exampled figure in Fig. V.3, we see that each branch of $\underline{s}[\delta v_1(o)]$ has, attached to each point on it, a displacement curve $\underline{t}[\delta v_1(o), \delta v_1(1)]$ with the corresponding value of $\delta v_1(o)$, sweeping off to ∞ as a function of $\delta v_1(1)$.

It seems quite likely that these curves will cover the whole space $\mathbb{R}^2$, in other words, there always exists a null-control sequence $\{\delta v_1(o), \delta v_1(1)\}$ for an arbitrary "error" state vector. These curves certainly cover a neighborhood of the origin in $\mathbb{R}^2$, since they are asymptotic to straight lines there. Here the relatively small values of $\delta v_1(o)$, $\delta v_1(1)$ imply only small curvature will be introduced by the $\delta v_1 \neq 0$ effect, as a multiplicative control variable, on the state-transition matrix.

But this picture also raises the question of how one could compute the accurate values of δv_1 for open-loop deadbeat control. The previous algorithm totally ignores the curvature of the covering loci and would be accurate only in the limit as $\delta v_1 \rightarrow 0$ near the origin in $\mathbb{R}^2$ (in eqn. (V.18) if $\underline{B}_{v_1} \delta v_1(o)$ and $\underline{B}_{v_1} \delta v_1(1)$ are ignored, $\underline{s}$ and $\underline{t}$ will be straight lines and are linear functions of $\delta v_1(o)$ and $\delta v_1(1)$ respectively, that is exactly how state-transition matrix $\tilde{\underline{M}}_j$ is set up in equation

(V.18)). That algorithm also involved computation of $\underline{\bar{\Phi}}^{-1}\underline{\delta X}(o)$ (or at least the bottom rows of all possible $\underline{\bar{\Phi}}$ matrices) either on-line or off-line (with bottom-row storage), which could be quite memory-demanding. An alternative that would account for the curvature effects as well, would be to store a grid-map of pairs of values $[\delta v_1(o), \delta v_1(1)]$ that cover a sufficiently large neighborhood of the origin in $\mathbb{R}^2$, then simply use the current measured value of $\underline{\delta X}(o)$ to look up the correct current control value. Actually, only $\delta v_1(o)$ values are needed with closed loop control, but they are needed for a two-dimensional (n-dimensional, in general) grid of points in $\mathbb{R}^2$. This can become equally demanding of memory. However, since if the grid has $\underline{p}$ points along each dimension in $\mathbb{R}^n$ space, the grid has $\underline{p}^n$ points, hence, we need $\underline{p}^n$ words of memory to store $\delta v_1(o)$ values. The previous algorithm requires n x n! words to store all possible bottom rows of $\underline{\bar{\Phi}}$. Using Stirling's formula:

$$n.n! \doteq n^{(n+1)} e^{-n} \sqrt{2\pi n} = n(\frac{n}{e})^n \sqrt{2\pi n}$$

we see that this will grow even faster than $\underline{p}^n$ for n large.

For n > 2, the loci covering arguments presented above can be extended in an obvious way to cover a neighborhood of the origin in R^n with "swept out" manifolds of growing dimension by families of parametric curves

$$\underline{s}[\delta v_1(o)]$$

$$\underline{s}^a + \underline{t}[\delta v_1{}^a(o), \delta v_1(1)]$$

$$\underline{s}^a + \underline{t}^b + \underline{r}[\delta v_1{}^a(o), \delta v_1{}^b(1), \delta v_1(2)]$$

etc.

Where $\underline{s}^a$ is a fixed point on the $\underline{s}(\delta v_1(o))]$ curve, $\underline{s}^a + \underline{t}^b$ is a fixed point on the $\underline{s} + \underline{t}$ surface and the family of all $\underline{r}$ displacements passing through all such fixed points covers a three-dimensional manifold, etc.

It is clear that these manifolds will always cover a neighborhood of the origin in R^n, since all the curves are asymptotic to the linear subspaces:

$$\tilde{\underline{M}}^{-1}_{i(o)} \underline{D}_{v_1}$$

$$\tilde{\underline{M}}^{-1}_{i(o)} \tilde{\underline{M}}^{-1}_{i(1)} \underline{D}_{v_1} + \tilde{\underline{M}}^{-1}_{i(o)} \underline{D}_{v_1}$$

$$\tilde{\underline{M}}^{-1}_{i(o)} \tilde{\underline{M}}^{-1}_{i(1)} \tilde{\underline{M}}^{-1}_{i(2)} \underline{D}_{v_1} + \tilde{\underline{M}}^{-1}_{i(o)} \tilde{\underline{M}}^{-1}_{i(1)} \underline{D}_{v_1} + \tilde{\underline{M}}^{-1}_{i(o)} \underline{D}_{v_1}$$

etc.

This is assured by the fact that the $\tilde{\underline{M}}$ matrices are designed to have relatively-orthogonal fast eigenvectors which, for example, make $\tilde{\underline{M}}^{-1}_{i(o)} \underline{D}_{v_1}$ and $\tilde{\underline{M}}^{-1}_{i(o)} \tilde{\underline{M}}^{-1}_{i(1)} \underline{D}_{v_1}$ not parallel or nearly parallel.

Basically, the deadbeat control approach, that has been talked about until now, is that assuming one of the multiplicative additive control inputs, say δv_1, is negligibly small so that a set of fast eigenvectors can be approximately obtained, and these fast eigenvectors are used to restore the system state back to its equilibrium point. The solutions to the problem of finding δv_1 are proposed, δv_1's are obtained either by inverting a matrix $\bar{\underline{\phi}}$, or by setting up a map of values of δv_1's in a multi-dimensional space.

Although this method looks promising, there are several problems, as follows, that remain to be solved. The global controllability has not been examined and it is doubtful that based on the richness of the system structure, we can obtain global controllability for a general bilinear system. In appendix IV, a particular bilinear system (the solar-assisted heat pump system) is examined in terms of its eigenvalues. As we see, the eigenvalues of this system are always negative, and this violates the sufficient conditions for complete controllability [2], although it is not enough for us to draw any conclusion yet, but it appears the system is hardly globally controllable.

The property of the set of the equilibrium points (eqn. V.12) is not investigated either. This should be done and hopefully, to be worthwhile, the equilibrium set will cover the whole state-space or at least a large portion of it.

There also exists the practical problem of implementing the controller. This approach can be referred to as supoptimal, nevertheless, nothing has been done to prove that it is nearly optimal. Neither is there any proof that we can always obtain a set of fast eigenvectors.

Due to the time limitation we have not been able to address every aspect of this approach, however, we believe that this approach could be feasible especially in solar-assisted systems.

REFERENCES of APPENDIX V

1. R.E. Rink, Private communication.

2. R.R. Mohler, "Bilinear Control Processes", Academic Press, New York, (1973).

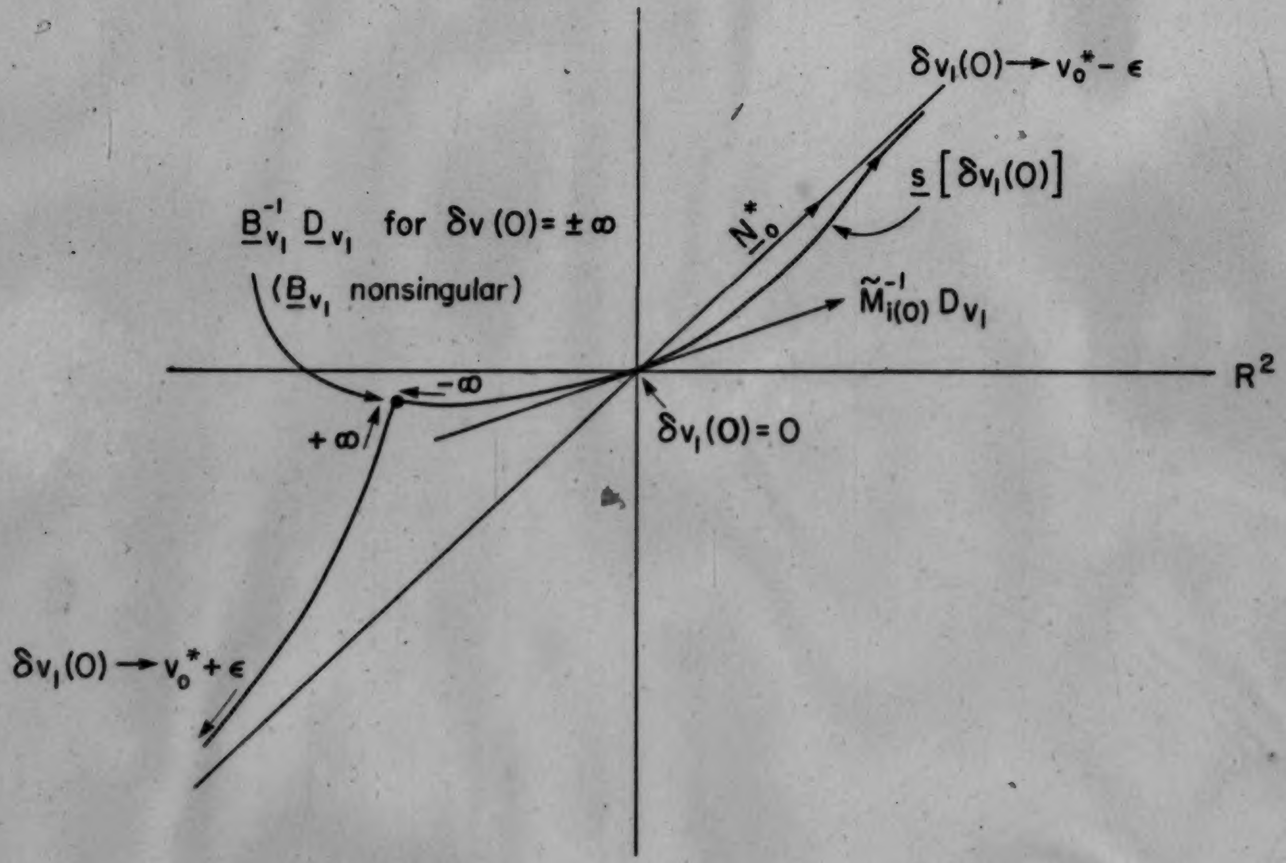
$\delta v_1(0) \rightarrow v_0^* - \epsilon$
$\underline{s}\,[\delta v_1(0)]$
$\underline{N}_0^*$
$\tilde{M}_{i(0)}^{-1} D_{v_1}$
R^2
$\underline{B}_{v_1}^{-1}\,\underline{D}_{v_1}$ for $\delta v(0) = \pm\infty$
($\underline{B}_{v_1}$ nonsingular)
$-\infty$
$+\infty$
$\delta v_1(0) = 0$
$\delta v_1(0) \rightarrow v_0^* + \epsilon$

Fig. V.1: Locus of s (one value v_0^*)

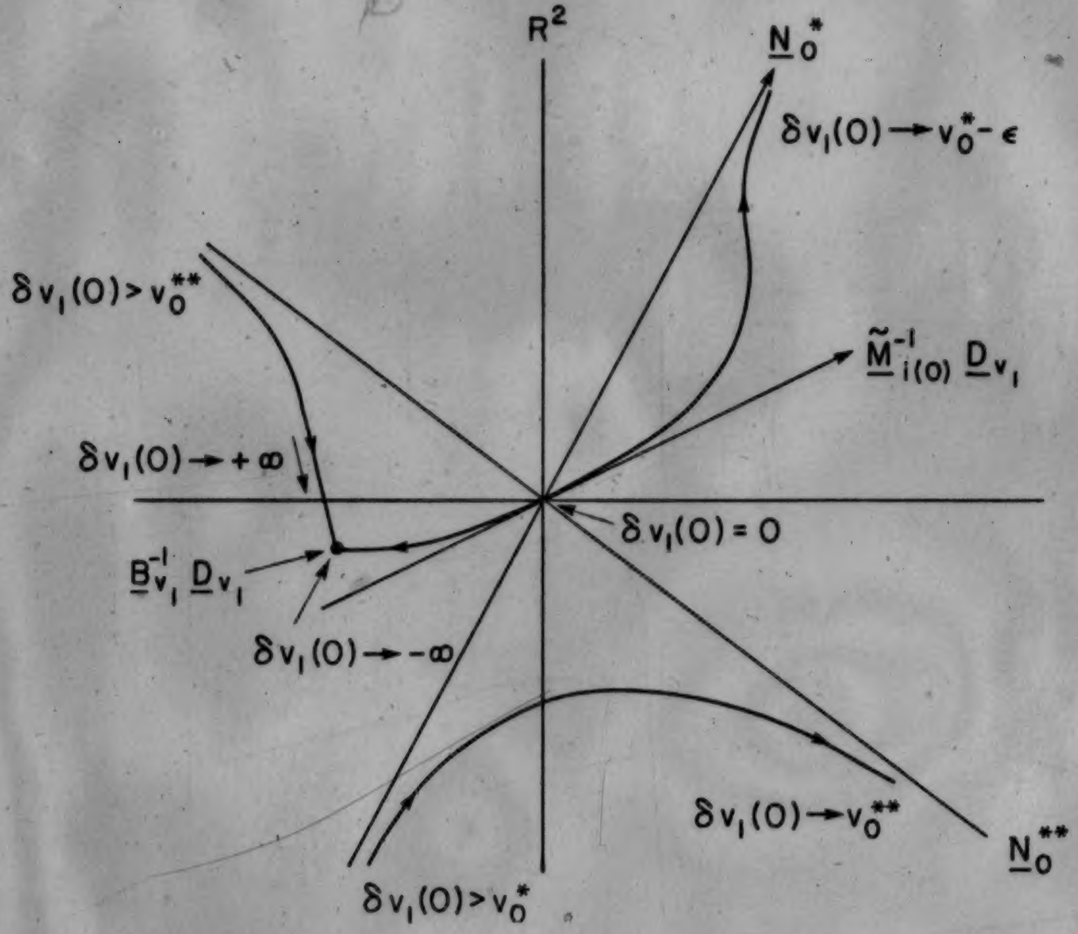
R^2
$\underline{N}_0^*$
$\delta v_1(0) \rightarrow v_0^* - \epsilon$
$\delta v_1(0) > v_0^{**}$
$\underline{\tilde{M}}_{i(0)}^{-1}\,\underline{D}_{v_1}$
$\delta v_1(0) \rightarrow +\infty$
$\delta v_1(0) = 0$
$\underline{B}_{v_1}^{-1}\,\underline{D}_{v_1}$
$\delta v_1(0) \rightarrow -\infty$
$\delta v_1(0) \rightarrow v_0^{**}$
$\underline{N}_0^{**}$
$\delta v_1(0) > v_0^*$

Fig. V.2: Locus of s (two value v_0^* and v_0^{**})

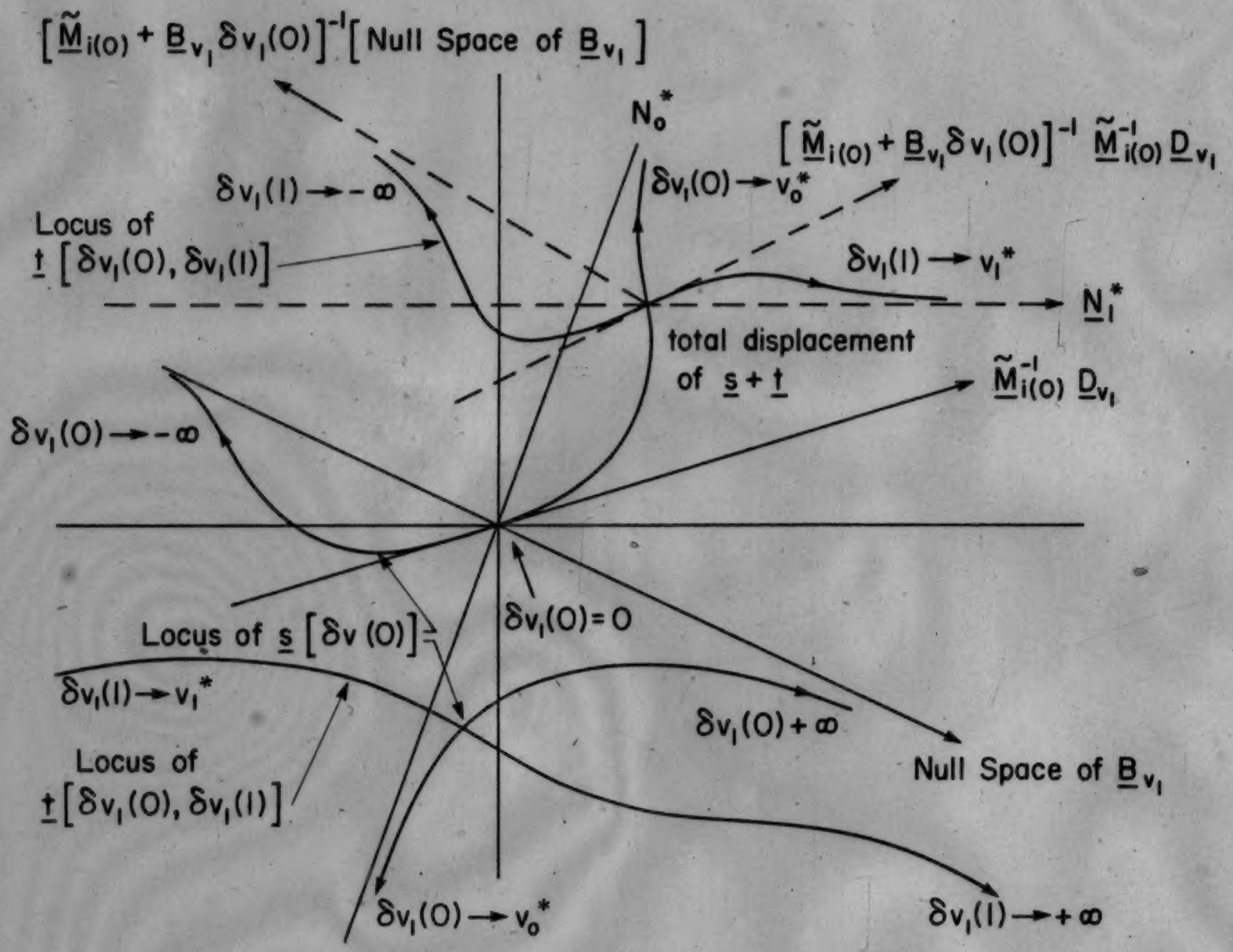
$[\underline{\tilde{M}}_{i(0)} + \underline{B}_{v_1}\delta v_1(0)]^{-1}$[Null Space of $\underline{B}_{v_1}$]
N_0^*
$[\underline{\tilde{M}}_{i(0)} + \underline{B}_{v_1}\delta v_1(0)]^{-1}\ \underline{\tilde{M}}^{-1}_{i(0)}\ \underline{D}_{v_1}$
$\delta v_1(1) \rightarrow -\infty$
$\delta v_1(0) \rightarrow v_0^*$
Locus of
$\underline{t}\ [\delta v_1(0), \delta v_1(1)]$
$\delta v_1(1) \rightarrow v_1^*$
$\underline{N}_1^*$
total displacement
of $\underline{s} + \underline{t}$
$\underline{\tilde{M}}^{-1}_{i(0)}\ \underline{D}_{v_1}$
$\delta v_1(0) \rightarrow -\infty$
Locus of $\underline{s}\ [\delta v(0)]$
$\delta v_1(0) = 0$
$\delta v_1(1) \rightarrow v_1^*$
$\delta v_1(0) + \infty$
Locus of
$\underline{t}[\delta v_1(0), \delta v_1(1)]$
Null Space of $\underline{B}_{v_1}$
$\delta v_1(0) \rightarrow v_0^*$
$\delta v_1(1) \rightarrow +\infty$

Fig. V.3: Locus of (s+t)

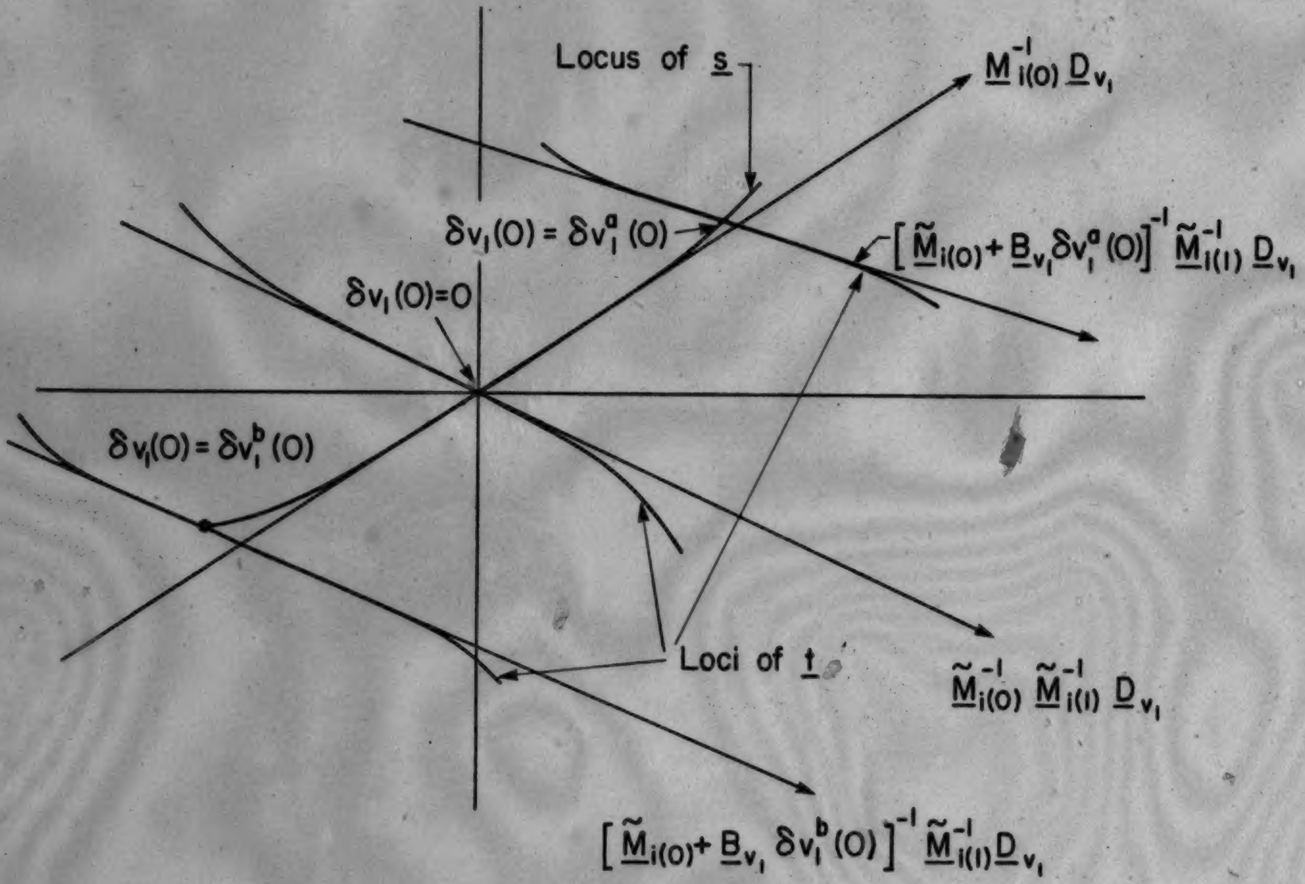
Locus of s
$\underline{M}^{-1}_{I(0)}\,\underline{D}_{v_1}$
$\delta v_1(0) = \delta v_1^a(0)$
$\left[\underline{\tilde{M}}_{I(0)} + \underline{B}_{v_1}\delta v_1^a(0)\right]^{-1}\underline{\tilde{M}}^{-1}_{I(1)}\,\underline{D}_{v_1}$
$\delta v_1(0)=0$
$\delta v_1(0) = \delta v_1^b(0)$
Loci of t
$\underline{\tilde{M}}^{-1}_{I(0)}\,\underline{\tilde{M}}^{-1}_{I(1)}\,\underline{D}_{v_1}$
$\left[\underline{\tilde{M}}_{I(0)} + \underline{B}_{v_1}\,\delta v_1^b(0)\right]^{-1}\underline{\tilde{M}}^{-1}_{I(1)}\underline{D}_{v_1}$

Fig. V.4: Locus of (s+t) in the neighborhood of the origin